THE

ELEMENTARY

ARITHMETIC.

Compiled and arranged by

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PREFACE.

The "Elementary Arithmetic" is intended to occupy an intermediate position, coming between the concrete and the advanced stages, and is adapted for the junior classes in our common schools, for securing the mental development, as well as the accuracy and expedition in calculation of the pupils between seven and eleven years of age.

The plan consists of such a delineation of the principles that the pupils are enabled, by induction, to form the appropriate rules.

After the accuracy of their knowledge is tested by a few mental exercises, the examples are reduced to practice on the blackboard or slate.

A number of self-testing exercises to many of the rules are introduced, which will save the teacher much labour, and be of benefit to the pupils.

The definitions and tables have been interspersed through the work, thereby rendering them more available to the student.

The plan pursued in the rule of Practice, is, we think, well calculated to exercise the reflective powers of the young, the examples and illustrations having been carefully selected, rising from the easy to the more difficult.

After Practice, Proportion is introduced, in a way not usually found in works of the kind; and several operations generally included under Interest and other rules, are grouped together, by which means the pupils are enabled to solve all questions where ratio is involved.

Under each rule will be found a large number of well graded exercises, many of which have been selected trom real occurrences in business.

The compiler has availed himself of the best works in the New and the Old World, viz., Dr. Robinson's, edited by Fish, Dr. Thomson's, Greenleaf's, Barnard Smith's, Currie's, Hay's and others, but especially that of Dr. Robinson.

NOTE.—In this Work, £ s. d. mean Sterling Money; $ and cts. mean Canada Currency.

THE ELEMENTARY ARITHMETIC.

DEFINITIONS.

1. Anything which can be *multiplied*, *divided* or *measured* is called QUANTITY. Thus, lines, weight, time, number, &c., are quantities.

2. Arithmetic is the science of number, and teaches how to represent numbers by symbols or signs, and the various methods of using these in calculation.

3. Numbers are expressions for one or more units. Thus, the words *one*, *two*, *three*, *four*, &c., or the characters 1, 2, 3, 4, &c., are expressions by which we indicate how many single things, or units, are to be taken.

4. Numbers are divided into two classes, **Abstract** and **Concrete** or DENOMINATE. If the units represented have no reference to any particular object, as when we say *seven* and *two* are *nine*, they are called abstract numbers. If the units referred to have reference to *particular* objects, as *two days*, *seven* men, &c., they are called *concrete* or *denominate* numbers.

NOTATION AND NUMERATION.

Art. 1. Notation is the *writing* or expressing of numbers by characters; and

Numeration is the *reading* of numbers expressed by characters.

2. Two systems of notation are in general use—the *Roman* and the *Arabic*.

The Roman Notation

3. Employs seven capital letters to express numbers. Thus,

Letters—	I	V	X	L	C	D	M
Values—	*one,*	*five,*	*ten,*	*fifty,*	*one hundred,*	*five hundred,*	*one thousand.*

By combining these letters, the ancient Romans formed the following

Table of Notation.

I ..	1	VIII ..	8	XV ..	15	XL ..	40
II ..	2	IX ..	9	XVI ..	16	L ..	50
III ..	3	X ..	10	XVII ..	17	LXX ..	70
IV ..	4	XI ..	11	XVIII ..	18	C ..	100
V ..	5	XII ..	12	XIX ..	19	D ..	500
VI ..	6	XIII ..	13	XX ..	20	M ..	1000
VII ..	7	XIV ..	14	XXX ..	30	MD ..	1500

This system of notation is principally confined to the numbering of chapters of books, public documents, &c.

Express the following numbers by letters:

1. Eleven.
2. Fifteen.
3. Seventeen.
4. Twenty-five.
5. Thirty-nine.
6. One thousand and one.
7. Ninety-nine thousand, four hundred.
8. One thousand, nine hundred and ten.
9. Express the present year.

The Arabic Notation

4. Employs ten characters, or figures, to express numbers. Thus,

Figures,	1	2	3	4	5	6	7	8	9	0
Names and values,	*one,*	*two,*	*three,*	*four,*	*five,*	*six,*	*seven,*	*eight,*	*nine,*	*nought or cipher.*

The first nine characters are called *significant figures,* because each has a value of its own. They are also called *digits,* a word derived from the Latin word *digitus,* which signifies *finger.*

The nought or cipher is also called *nothing* or *zero.* The cipher has, of itself, no value, but is used to indicate the order of the significant figures which precede it.

The ten Arabic characters are the Alphabet of Arithmetic; and by combining them according to certain principles, all numbers can be expressed.

5. To facilitate the reading of large numbers they are divided into periods of three figures each, beginning at the right-hand side, according to the following

Numeration Table.

Period	I.	Units	1	Units,
			2	Tens,
			3	Hundreds.
"	II.	Thousands....	4	Units of Thousands,
			5	Tens of Thousands,
			6	Hundreds of Thousands,
"	III.	Millions.......	7	Units of Millions,
			8	Tens of Millions,
			9	Hundreds of Millions.
"	IV.	Billions	10	Units of Billions,
			11	Tens of Billions,
			12	Hundreds of Billions.
"	V.	Trillions......	13	Units of Trillions,
			14	Tens of Trillions,
			15	Hundreds of Trillions.
"	VI.	Quadrillions ...	16	Units of Quadrillions,
			17	Tens of Quadrillions,
			18	Hundreds of Quadrillions.

6. Figures occupying different places in a number, as units, tens, hundreds, &c., are said to express different orders of units.

Simple units are called units of the			*first*	order.
Tens	"	"	*second*	"
Hundreds	"	"	*third*	"
Thousands	"	"	*fourth*	"

and so on. Thus, 327 contains 3 units of the third order, 2 units of the second order, and 7 units of the first order.

Exercises for the Slate.

Write and read the following numbers:

1. One unit of the third order, four of the second.
2. Eight units of the fifth order, three of the second.
3. Two units of the seventh order, five of the sixth, three of the fourth, nine of the third, eight of the first.

4. Four units of the tenth order, six of the eighth, four of the seventh, three of the fifth, seven of the fourth, nine of the second, one of the first order.

7. Principles of Notation and Numeration.

1st. Figures have two values, Simple and Local.

The **Simple Value** of a figure is its value when taken alone. Thus, 3, 4, 5.

The **Local Value** of a figure is its value when used with another figure or figures in the same number. Thus, in 472 the simple values of the several figures are 4, 7, and 2; but the local value of the 4 is 400; of the 7 is 7 tens, or 70; and of the 2 is 2 units.

NOTE.—When a figure occupies the first place, its simple and local values are the same.

2nd. A digit or figure, if used in the second place, expresses tens; in the third place, hundreds; in the fourth place, thousands; and so on.

3rd. As 10 units make 1 ten, 10 tens 1 hundred, 10 hundreds 1 thousand, and 10 units of any order, or in any place, make 1 unit of the next higher order, we readily see that the Arabic form of notation is based on the following

GENERAL LAWS.

I. The different orders of units increase from right to left, in a ten-fold ratio.

II. Every removal of a figure one place to the left, increases its local value ten-fold; and every removal of a figure one place to the right, diminishes its local value to one-tenth of its previous value. Thus,

6 is 6 units.
60 is 10 times 6 units.
600 is 10 times 6 tens.
6000 is 10 times 6 hundreds.

4th. Every period contains three figures, (units, tens, and hundreds,) except the left hand period, which sometimes contains only one or two figures, (units, or units and tens.)

RULE FOR NOTATION.

I. Beginning at the left hand, write the figures belonging to the highest period.

II. Write the hundreds, tens, and units of each successive period in their order, placing a cipher wherever an order of figures is wanting.

RULE FOR NUMERATION.

I. Separate the number into periods of three figures each, commencing at the right hand.

II. Beginning at the left hand, read off the number of units of each order in each period separately, and add the name of the period.

NOTE.—In reading numbers the name of the last, or right-hand period, is usually omitted.

8. Until the pupil can write numbers readily, it may be well for him to write several periods of ciphers, point them off, and over each period write its name. Thus,

Trillions,	Billions,	Millions,	Thousands,	Units.
0 0 0 ,	0 0 0 ,	0 0 0 ,	0 0 0 ,	0 0 0

And then write the given numbers in their appropriate places.

Exercises for the Slate.

Express the following numbers by figures:

1. Thirty-six.
2. Three hundred and thirty-six.
3. Five thousand, three hundred and thirty-six.
4. Fourteen thousand, two hundred and forty-seven.
5. Four hundred and fifty thousand, and fifty-nine.
6. Ninety-six thousand and four.
7. Nine hundred thousand, and ninety.
8. Sixty-one billions, four millions, and ninety-seven.

Point off, and read the following numbers:

9.	489	14.	3786	19.	2987654300
10.	586	15.	20900	20.	4783006001
11.	4070	16.	57631	21.	3456789012
12.	307	17.	37000	22.	6830428301
13.	10010	18.	94000554	23.	7932643162

24. Write seven millions and thirty-six.
25. What orders of units are contained in the number 10370500?

ADDITION.

Explanatory Exercises.

9. 1. John gave 5 dollars for a vest, and 9 dollars for a coat; how many dollars did he pay for both?

Analysis.—He gave as many dollars as 5 dollars and 9 dollars, which are 14 dollars.

2. A farmer sold a lamb for 3 dollars, and a calf for 4 dollars; how many dollars did he receive for both?

3. John got 3 apples from his mother, 2 apples from his sister, and 1 apple from his brother; how many apples did he get altogether?

4. How many are 4 and 5? 4 and 7? 3 and 6?

5. How many are 5 cents, 6 cents, and 7 cents?

10. From the preceding operations we perceive that **Addition** is the process of uniting *several* numbers into *one* equivalent number.

11. The **Sum** or **Amount** is the result obtained by the process of addition.

Note.—Concrete numbers, that is *numbers of objects*, cannot be added together unless the objects are of the same kind. Thus, 4 grammars and 5 geographies cannot be added together. If, however, we drop the distinctive names of the objects, and use in their stead a more general term, which will include the several kinds in one class, the addition can be performed. Thus, if we consider geographies and grammars merely as *books*, we may say 4 grammars (books) and 6 geographies (books) are 10 books. This principle applies to all operations with concrete numbers.

12. The sign +, is called *plus*, which signifies *more*. When placed between two numbers, it denotes that they are to be added together. Thus, 6 + 3, shows that 3 is to be added to 6.

CASE I.

13. *When the amount of each column is less than* 10.

Example 1.—A farmer sold a horse for 103 dollars, seven cows for 271 dollars, and some hay and oats for 124 dollars; how much did he receive for all?

OPERATION.

	Hundreds.	Tens.	Units.
	1	0	3
	2	7	1
	1	2	4
Amount	4	9	8

Analysis.—We arrange the numbers so that units of like order shall stand in the same column. We then add the columns separately, for convenience commencing at the right hand, and write each result under the column added. Thus, we have 4 and 1 and 3 are 8, the sum of the units; 2 and 7 are 9, the sum of the tens; 1 and 2 and 1 are 4, the sum of the hundreds. Hence, the entire amount is 4 hundreds 9 tens and 8 units, or 498, the Answer.

Exercises for the Slate.

SECTION I.

1. Dollars.	2. Miles	3. Cents.	4. Days.
172	437	361	245
116	140	227	321
101	321	410	132
Ans. 389			

5. What is the sum of 126, 321 and 232? Ans. 679.
6. What is the amount of 521, 142 and 231? Ans. 894.
7. A stock farmer bought three droves of sheep. The first contained 225, the second 301, and the third 463; how many sheep did he buy in all? Ans. 989.

CASE II.

14. *When the amount of any column equals or exceeds* 10.

EXAMPLE 2.—A gentleman pays 596 dollars a year for house rent, 366 dollars for servants' wages, and 989 dollars for other expenses; what is the amount of his expenses?

OPERATION.

		Hundreds.	Tens.	Units.
		5	9	6
		3	6	6
		9	8	9
Sum of the units			2	1
Sum of the tens		2	3	
Sum of the hundreds	1	7		
Total amount	1	9	5	1

ANALYSIS.—Arranging the numbers as in Case I, we first add the column of units, and find the sum to be 21 units. We write the 1 unit in the place of units, and the two tens in the place of tens. The sum of the figures in the column of tens is 23 tens, which is 2 hundreds, and 3 tens. We write the 3 tens in the place of tens, and the two hundreds in the place of hundreds.—We next add the column of hundreds, and find the sum to be 17 hundreds, which is 1 thousand and 7 hundreds. We write the 7 hundreds in the place of hundreds, and 1 thousand in the place of thousands. Lastly, by uniting the sum of the units with the sum of the tens and hundreds, we find the total amount to be 1 thousand 9 hundreds 5 tens and 1 unit, or 1951.

This example may be performed by another method, which is the one in common use. Thus,

OPERATION.
596
366
989
1951

ANALYSIS.—Arranging the numbers as before, we add the first column and find the sum to be 21 units; writing the 1 unit under the column of units, we add the two tens to the column of tens, and find the sum to be 25 tens; writing the 5 tens under the column of tens, we add the two hundreds to the column of hundreds, and find the sum to be 19 hundreds; as this is the last column, we write down its amount, 19, and we have the *whole amount*, 1951, as before.

NOTE.—Units of the same order are written in the same column; and when the sum in any column is 10, or more than 10, it produces *one or more units* of a higher order, which must be added to the next column. This process is sometimes called "carrying the tens."

15. From the preceding examples and illustrations we deduce the following

RULE. I. Write the numbers to be added so that all the units of the same order shall stand in the same column; that is, units under units, tens under tens, &c.

II. Commencing at units, add each column separately, and write the sum underneath, if it be less than ten.

III. If the sum of any column be ten, or more than ten, write the unit figure only, and add the ten or tens to the next column.

IV. Write the entire sum of the last column.

Mental Exercise.

1. How many are 6 and 7? 6 and 9? 6 and 13?
2. How many are 6 units, 9 tens, and 15 units?
3. How many are 8 dollars, and 13 dollars, and 15 dollars?
4. How many are $6 + 7 + 8 + 9 + 12 + 13 + 8$?
5. A man gave 12 dollars for some oats, 8 dollars for a ton of hay, and 7 dollars for a barrel of flour; how many dollars did he pay for all?
6. A man bought a sleigh for 26 dollars, paid 10 dollars for lining it and 11 dollars for painting it; what did it cost him?
7. A tailor bought three pieces of cloth, the first containing 29 yards, the second 27 yards, and the third 42; how many yards did the three pieces contain?
8. A man bought a barrel of flour for 7 dollars and sold it so as to gain 3 dollars; how much did he sell it for?

Exercises for the Slate.

NOTE.—All the Exercises for the slate, given in this work, which have not the answers at ached are self-testing, the Key to which may be found in the appendix.

SECTION II.

(1)	(2)	(3)	(4)
3456	4563	5787	35109
3456	4563	5787	35109
6912	9126	11574	70218
10368	13689	17361	105327
17280	22815	28935	175545

(5)	(6)	(7)	(8)
67896	24687	84906	54639
67896	24687	84906	54639
135792	49374	169812	109278
203688	74061	254718	163917
339480	123435	424530	273195

16. The sign $=$, is called the sign of *equality*. When placed between two numbers, or sets of numbers, it signifies that they are equal to each other. Thus, the expression $6 + 4 = 10$, is read 6 *plus* 4 is *equal to* 10, and denotes that the numbers 6 and 4 taken together, equal the number 10.

SECTION III.

In the following exercises take the given number for the first and second lines or rows, their sum for the third, the sum of the third and second for the fourth, and so on, adding the last two for the next row. Finally, add the whole.

NOTE.—5 r. means 5 rows, 6 r. means 6 rows, &c.

EXAMPLE.—What is the sum of 3456 extended to 5 rows.

OPERATION.

First row	3456	
Second "	3456	Same as first row.
Third "	6912	= Sum of second and first.
Fourth "	10368	= Sum of third and second.
Fifth "	17280	= Sum of fourth and third.
Ans.	41472	= Sum of all the rows.

	6 r.		6 r.		6 r.		6 r.
(1)	63	(8)	171	(15)	1233	(22)	109872
(2)	72	(9)	621	(16)	4581	(23)	234531
(3)	45	(10)	531	(17)	6543	(24)	901827
(4)	54	(11)	432	(18)	7632	(25)	728109
(5)	27	(12)	135	(19)	8901	(26)	879102
(6)	36	(13)	252	(20)	9342	(27)	512361
(7)	18	(14)	801	(21)	1899	(28)	987642

(29)	632781	(34)	1234584	(39)	240357897
(30)	547182	(35)	2781099	(40)	304578927
(31)	987606	(36)	3765789	(41)	457028973
(32)	875871	(37)	4572171	(42)	758203434
(33)	767808	(38)	5706018	(43)	987645312

SHOW THAT

(1)	45	extended	8 r.	=	18	extended	8 r.	+	27	extended	8 r.
(2)	54	"	8 r.	=	36	"	8 r.	+	18	"	8 r.
(3)	153	"	6 r.	=	90	"	6 r.	+	63	"	6 r.
(4)	162	"	6 r.	=	72	"	6 r.	+	90	"	6 r.
(5)	549	"	5 r.	=	261	"	5 r.	+	288	"	5 r.
(6)	1089	"	4 r.	=	531	"	4 r.	+	558	"	4 r.

SECTION IV.

1. Find the sum of 1247 + 91679 + 27 + 1987 + 1800 1796. Ans. 98536.

2. What is the sum of 250120 + 30402 + 7850 + 465000 + 10046 + 65045. Ans. 828463.

3. Add together 786, 840, 910, 403, 783, 650, 809, 670 408, 310, and 652. Ans. 7221.

4. Add together 16075, 250763, 7561, 830654, 293106, 2537104, and 316725. Ans. 4251988.

5. Find the sum of 629405, 7629, 31000401, 263012, 1300512, 390217, and 13268. Ans. 33604444.

6. A man gave 5460 dollars to his eldest son, to the next 4065, to the next 6750, to the next 8000, and to the youngest 7276; how much did he give to all. Ans. 31551 dollars.

7. A merchant on settling up his business, found he owed one creditor 176 dollars, another 841 dollars, another 1356 dollars, another 2370 dollars, another 840 dollars; what was the amount of his debts? Ans. 5583 dollars.

8. Find the sum of the following numbers: seven hundred and fifty-six, four hundred and twenty-five, six hundred and

thirty-three, five hundred and forty-one, nine hundred and sixty-nine. Ans. 3324.

9. Add together six, sixty-five, six hundred and fifty-five, three thousand six hundred and fifty-five, twenty-six thousand three hundred and fifty-nine. Ans. 30740.

10. A man willed his estate to his wife, two sons, and four daughters. To his daughters he gave 2630 dollars apiece, to his sons, each 4647 dollars, and to his wife 3595 dollars; of what value was his estate? Ans. 23409 dollars.

11. A man bought three houses and lots for 15780 dollars, and sold them so as to gain 695 dollars on each lot; for how much did he sell them? Ans. 17865 dollars.

SUBTRACTION.

Explanatory Exercises.

17. A farmer having 8 cows, sold 3 of them, how many cows had he left?

Analysis.—He had as many left as 8 cows less 3 cows, which are 5 cows. Therefore he had 5 cows left.

2. David has 9 peaches, and George has seven peaches; how many more peaches has David than George?

Analysis.—Here, as in the former case, he has as many more as 9 peaches less 7, which are 2 peaches. Therefore he has 2 peaches more than George,

3. A merchant having 14 barrels of flour, sells nine of them; how many has he left?

4. Paid 19 dollars for a coat, and 4 dollars for a vest; how much more did the coat cost than the vest?

18. We see from the foregoing that **Subtraction** is the process of determining the difference between two numbers.

19. The **Minuend** is the number to be subtracted from.

20. The **Subtrahend** is the number to be subtracted.

21. The **Difference** or **Remainder** is the result obtained by the process of subtraction.

22. The sign —, is called *minus*, which signifies *less*. When placed between two numbers, it denotes that the one

after it is to be taken from the one before it. Thus, 7 — 3, is read 7 *minus* 3, and means that 3 is to be taken from 7.

CASE I.

23. *When no figure in the subtrahend is greater than the corresponding figure in the minuend.*

EXAMPLE 1.—From 697 take 432.

OPERATION.	
Minuend	697
Subtrahend	432
Remainder	265

ANALYSIS.—We write the less number under the greater, with units under units, tens under tens, &c., and draw a line underneath. Then, beginning at the right hand, we subtract separately each figure of the subtrahend from the figure above it in the minuend. Thus, 2 from 7 leaves 5, which is the difference of the units; 3 from 9 leaves 6, the difference of the tens; 4 from 6 leaves 2, the difference of the hundreds. Hence, we have for the whole difference 2 hundreds, 6 tens, and 5 units, or 265.

Exercises for the Slate.

SECTION I.

	(1)	(2)	(3)	(4)
Minuend	543	876	367	978
Subtrahend	212	334	152	725
Remainder	331	542	215	253

		Remainders.
5.	From 98765 take 74251	24514
6.	From 291352 take 170341	121011
7.	Subtract 291352 from 895752	604400

8. A man bought a property for 3724 dollars, and sold it for 4856 dollars; how much did he gain? Ans. 1132 dollars.

9. A drover bought 1598 sheep, and sold 473 of them; how many had he left? Ans. 1125 sheep.

10. A merchant sold flour to the amount of 6578 dollars, and by so doing gained 2426 dollars; how much did he pay for the flour? Ans. 4152 dollars.

CASE II.

24. *When any figure in the subtrahend is greater than the corresponding figure in the minuend.*

EXAMPLE 1.—From 846 take 359.

OPERATION.

Hundreds.	Tens.	Units.
8	4	6
3	5	9
4	8	7

ANALYSIS.—Since we cannot take 9 units from 6 units, we add 10 units to 6 units, making 16 units; 9 units from 16 units leave 7 units. But as we added 10 units, or 1 ten, to the minuend, we have a remainder 1 ten too large, to balance which, we add 1 ten to the five tens in the subtrahend, making 6 tens. We cannot take 6 tens from 4 tens; so we add 10 tens to 4 making 14 tens; 6 tens from 14 tens leaves 8 tens. Now having added 10 tens, or 1 hundred, to the minuend, we have a remainder 1 hundred too large, to balance which we add 1 hundred to the 3 hundreds in the subtrahend, making 4 hundreds; 4 hundreds from 8 hundreds leave 4 hundreds, and we have for the total remainder, 487.

NOTE.—The process of adding 10 to the minuend is sometimes called *borrowing* 10, and that of adding 1 to the next figure of the subtrahend, *carrying* one.

25. From the preceding examples and illustration we have the following general

RULE. I. Write the less number under the greater, placing units of the same order in the same column.

II. Begin at the right hand, and take each figure of the subtrahend from the figure above it, and write the result underneath.

III. If any figure in the subtrahend be greater than the corresponding figure above it, add 10 to that upper figure before subtracting, and then add 1 to the next left hand figure of the subtrahend.

Mental Exercises.

1. A man, having 25 dollars due him, received a ton of hay worth 11 dollars, and the remainder in money; how much money did he receive?

2. A farmer sold a cow for 23 dollars, that cost him 31 dollars; how much did he lose by the bargain?

3. From a piece of broadcloth containing 72 yards, 26 yards were cut; how many yards remained?

4. A boy found 8 apples under one tree, 10 under another, and 6 under another; he ate 4, gave away 6, and carried the remainder home; how many did he take home?

5. A farmer had 43 sheep in one lot, 39 in another, and 40 in another; from the first he sold 20, from the second 15,

and from the third 17; how many had he at first, and how many had he left?

Exercises for the Slate.

SECTION II.

(1)	(2)	(3)	(4)
203688	10368	13689	17361
135792	6912	9126	11574

(5)	(6)	(7)	(8)
74061	254718	163917	2367468
49374	169812	109278	1578312

(9)	18717— 12478	(16)	239596137—159730758
(10)	703701— 469134	(17)	243401058—162267372
(11)	1037016— 691344	(18)	272729889—181819926
(12)	1281933— 854622	(19)	111056292— 74037528
(13)	6131016— 4087344	(20)	259237071—172824714
(14)	2017035— 1344690	(21)	16931349— 11287566
(15)	2412072— 1608048	(22)	19313505— 12875670

SECTION III.

1. From 7238469153 take 4298376593.
Ans. 2940092560

2. From 9758354961 take 4938297562.
Ans. 4820057399.

3. From 9738426549 take 9423689284.
Ans. 314737265

4. Take 6428395823 from 9035482762.
Ans. 2607086939.

5. Take 729384 from 920376842.
Ans. 919647458.

6. From 9784 + 3968, take 3268 + 5274.
Ans. 5210.

7. From 8764 + 398 + 41, take 39 + 481 + 6324.
Ans. 2359.

8. A man owning a block of buildings worth 155265 dollars, keeps it insured for 109240 dollars; how much would he lose in case the buildings should be destroyed by fire?
Ans. 46025 dollars.

9. A merchant paid 17894 dollars for a steamboat, and

The operation in this example may be performed in another way, which is the one in common use.

OPERATION.
484
4
1936

ANALYSIS.—Writing the numbers as before, we begin at the right hand or unit figure, and say: 4 times 4 units are 16 units, which is 1 *ten* and 6 *units;* write the 6 units in the product in units' place, and reserve the 1 ten to add to the next product. 4 times 8 tens are 32 tens, and the 1 ten reserved in the last product added, are 33 tens, which is 3 *hundreds* and 3 *tens;* write the 3 tens in the product in tens' place, and reserve the 3 hundreds to add to the next product. 4 times 4 hundreds are 16 hundreds, and 3 hundreds added are 19 hundreds, which being written in the product in the places of hundreds and thousands, gives, for the entire product, 1936.

34. From the preceding example and illustration we have the following

RULE. I. Write the multiplier under the multiplicand, placing units of the same order under each other.

II. Beginning with the unit figure multiply each figure of the multiplicand by the multiplier, writing down and carrying as in addition.

Mental Exercises.

1. If a man can dig 28 bushels of potatoes in one day; how many can he dig in 7 days? in 9 days? in 12 days?
2. At 81 dollars apiece, what will be the cost of 4 horses? of 11 horses? of 9 horses?
3. In an orchard there are 16 cherry trees, and 9 times as many apple trees; how many apple trees are there?
4. If one boy earns 15 cents a day, another 22 cents a day, and another 30 cents a day; how much can the 3 boys earn in 5 days?
5. A man bought 9 yards of cloth for a suit of clothes, at 6 dollars a yard: he paid 5 dollars for making the coat, 2 dollars for making the pantaloons, and 1 dollar for making the vest; what did the suit cost him?

Exercises for the Slate.

SECTION I.

1. Multiply 543216573 by 2, 3, 4, 5, 6, 7
2. Multiply 345678921 by 9, 8, 7, 6, 11.

Verify the following—

(3) $47 \times 2 = 19 \times 2 + 28 \times 2$	(7) $369 \times 2 = 246 \times 2 + 123 \times 2$
(4) $59 \times 2 = 27 \times 2 + 32 \times 2$	(8) $663 \times 2 = 431 \times 2 + 232 \times 2$
(5) $75 \times 2 = 49 \times 2 + 26 \times 2$	(9) $984 \times 2 = 615 \times 2 + 369 \times 2$
(6) $124 \times 2 = 56 \times 2 + 68 \times 2$	(10) $196 \times 2 = 94 \times 2 + 102 \times 2$

NOTE.—Instead of 2 as multiplier take successively 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 as multipliers, using the exercises in the section.—Thus,

(10) $196 \times 9 = 94 \times 9 + 102 \times 9$, &c.

11. What will be the cost of 344 cords of wood at 4 dollars a cord? Ans. 1376 dollars.

12. In one day are 86400 seconds; how many seconds in 7 days? Ans. 604800 seconds.

13. In one bushel there are 256 gills; how many gills are there in 12 bushels? Ans. 3072 gills

CASE II.

35. *When the multiplier is a composite number, none of whose factors are greater than* 12.

36. A **Composite Number** is one that may be produced by multiplying together two or more numbers. Thus, 18 is a composite number, since $6 \times 3 = 18$; or $9 \times 2 = 18$; or $3 \times 3 \times 2 = 18$.

37. The **Component Factors** of a number are the several numbers which, multiplied together, produce the given number. Thus, the component factors of 16 are 4 and 4, $(4 \times 4 = 16)$; or, 8 and 2, $(8 \times 2 = 16)$; or, 2 and 2 and 2 and 2, $(2 \times 2 \times 2 \times 2 = 16)$.

NOTE.—The pupil must not confound the *factors* with the *parts* of a number. Thus, the *factors* of which 14 is composed are 7 and 2, $(7 \times 2 = 14)$; while the *parts* of which 14 is composed are 8 and 6 $(8 + 6 = 14)$, or, 10 and 4, $(10 + 4 = 14)$. The *factors* are *multiplied*, while the *parts* are *added*.

EXAMPLE 2.—What will 36 cows cost, at 196 dollars each?

Multiplicand	196	cost of 1 cow.
1st factor	4	
	784	cost of 4 cows.
2nd factor	9	
Product	7056	cost of 36 cows.

ANALYSIS.—The factors of 36 are 4 and 9. If we multiply the cost of 1 cow by 4, we obtain the cost of 4 cows; and by multiplying the cost of 4 cows by 9, we obtain

the cost of 9 times 4 cows, or 36 cows, the number bought. Hence we have the following

RULE. I. Separate the composite number into two or more factors.

II. Multiply the multiplicand by one of these factors, and that product by another, and so on until all the factors have been used successively, the last product will be the product required.

SECTION II.

Find the product of—

(1) 1236456 × 15
(2) 2345679 × 16
(3) 4571325 × 18
(4) 7235469 × 21
(5) 9876519 × 24
(6) 8297568 × 27
(7) 9726354 × 35
(8) 87645231 × 32
(9) 18765432 × 35
(10) 33236775 × 36
(11) 21876543 × 42
(12) 54670104 × 44
(13) 32336775 × 54
(14) 68206986 × 55

15. What will 573 oxen cost, at 63 dollars each?
Ans. 36099 dollars.

16. If an army consume 1645 pounds of bread in a day, how much will they consume in 96 days?
Ans. 157920 pounds.

17. How many are 84 times six hundred and four thousand, seven hundred and fifty-six? Ans. 50799504.

18. A merchant bought 145 pieces of broadcloth, each piece containing 48 yards, at 4 dollars a yard; how much did the whole cost? Ans. 27840 dollars.

CASE III.

38. *When the multiplier consists of two or more figures.*

EXAMPLE 3.—Multiply 646 by 29.

Multiplicand	646	
Multiplier	29	
	5814	9 times the multiplicand.
	1292	20 times the multiplicand.
Product	18734	29 times the multiplicand.

ANALYSIS.—Writing the multiplicand and multiplier as in Case I, we first multiply each figure of the multiplicand by the unit figure of the multiplier, exactly as in Case I. We then multiply by the 2 tens. 2 tens times 6 units, or 6 times 2 tens, are 12 tens, equal to 1 hundred, and 2 tens; we place the two tens

under the tens' place in the product already obtained. 2 tens times 4 tens are 8 hundreds, and 1 hundred of the last product added are 9 hundreds; we write the 9 under the hundreds' place in the product. 2 tens times 6 hundreds are 12 thousands, equal to 1 ten thousand and 2 thousands, which we write in their appropriate places in the product. Then adding the two products we have the entire product, 18734.

NOTE.—1. When the multiplier contains two or more figures, the several products obtained by multiplying by each figure are called *partial* products.

2. When there are ciphers between the significant figures of the multiplier, pass over them and multiply by the significant figures only.

39. From the preceding examples and illustrations we deduce the following general

RULE. I. Write the multiplier under the multiplicand, placing units of the same order under each other.

II. Multiply the multiplicand by each figure of the multiplier successively, beginning with the unit figure, and write the first figure of each partial product under the figure of the multiplier used, writing down and carrying as in Addition.

III. If there are partial products, add them, and their sum will be the product required.

40. *When there are ciphers at the right hand of one or both the factors.*

RULE. Multiply the significant figures of the multiplicand by those of the multiplier, and to the product annex as many ciphers as there are on the right of both factors.

SECTION III.

Multiply and add together the products of—

(1) 1678583214 by 701 and 299	(6) 912837654 by 827 and 173
(2) 7843221567 by 679 and 321	(7) 764583912 by 531 and 469
(3) 8976510234 by 348 and 652	(8) 837654219 by 204 and 796
(4) 2190678093 by 959 and 41	(9) 376542198 by 304 and 696
(5) 3672815490 by 869 and 131	(10) 6354819027 by 801 and 199

SECTION IV.

EXAMPLE.—546372 × 47 = 546372 × 19 + 546372 × 28. Thus,

546372	546372	546372
47	19	28
3824604	4917348	4370976
2185488	546372	1092744
25679484	10381068	15298416
	15288416	
	25699484	

Work the following as the preceding example—

(1)	87654321	× 14 =	87654321	×	6	+	87654321	×	8
(2)	13254876	× 19 =	13254876	×	8	+	13254876	×	11
(3)	54312786	× 25 =	54312786	×	12	+	54312786	×	13
(4)	65784123	× 37 =	65784123	×	18	+	65784123	×	19
(5)	73214658	× 49 =	73214658	×	24	+	73214658	×	25
(6)	83146752	× 65 =	83146752	×	39	+	83146752	×	26
(7)	90765639	× 104 =	90765639	×	39	+	90765639	×	65
(8)	75697281	× 143 =	75697281	×	88	+	75697281	×	55
(9)	79865379	× 592 =	79865379	×	286	+	79865379	×	306
(10)	57763323	× 111 =	57763323	×	99	+	57763323	×	12

SECTION V.

Divide each of the following exercises into *two* periods of three figures each, use these as multipliers, and test the results as in the following example:

134865 thus divided gives the multipliers 134, 865, then

134865 × 134 =	18071910
134865 × 865 =	116658225
Sum of products	134730135
The multiplicand	134865
Sum of products and multiplicand	134865000 = 1000 times the multiplicand.

(1)	134865	(11)	309690	(21)	892107
(2)	296703	(12)	327672	(22)	807192
(3)	237762	(13)	427572	(23)	735264
(4)	380619	(14)	456543	(24)	702297
(5)	523476	(15)	502497	(25)	586413
(6)	491508	(16)	617382	(26)	475524
(7)	357642	(17)	694305	(27)	486513
(8)	463536	(18)	264735	(28)	390609
(9)	375624	(19)	763236	(29)	420579
(10)	705294	(20)	789210	(30)	614385

SECTION VI.

1. What is the product of 71476 × 9187? Ans. 656650012.

2. Multiply 8010700 by 9000909. Ans. 72103581726300.

3. In 1 mile there are 63360 inches; how many inches in 45 miles? Ans. 2851200.

4. If in one year there are 8766 hours; how many hours in 72 years? Ans. 631152 hours.

5. What cost 97 oxen at 29 dollars each? Ans. 2813 dollars.

6. If a person deposit annually in the Savings' Bank 407 dollars; what will be the sum deposited in 27 years? Ans. 10989 dollars.

7. Multiply 875946 by 807004 Ans. 706891925784.

8. Multiply 948657 by 908070. Ans. 861446961990.

9. Multiply 496783 by 4263. Ans. 2117785929.

10. If a hogshead of sugar contains 1096 pounds; how many pounds in 27 hogsheads? Ans. 29592 pounds.

11. Find the continued product of 186, 396 and 56. Ans. 4124736.

12. Multiply eight thousand and nine by nine thousand and sixteen. Ans. 72209144.

13. Multiply one million one thousand one hundred by nine thousand nine hundred and ninety. Ans. 10000989000.

14. If a railroad car moves 38 miles an hour; how far would it go in 30 days, of 24 hours each, allowing 2 hours each day for stopping? Ans. 25080 miles.

15. If 9 men can do a piece of work in 13 days; how long would it take one man to do the same work? How many men would do it in one day? Ans. 117 days. 117 men.

16. A merchant bought 563 barrels of shoe pegs, each barrel containing 4 bushels, at 5 shillings a bushel; how many shillings did he give for the whole? Ans. 11260 shillings.

DIVISION.

Explanatory Exercises.

41. 1. A boy has 32 cents which he wishes to give to 4 of his companions, to each an equal number; how many cents must each receive?

ANALYSIS.—Since there are four companions each must receive as many cents as 4 is contained times in 32, which is 8 times. Therefore, each boy will receive 8 cents.

2. How many barrels of flour, at 8 dollars per barrel, can you buy for 56 dollars?

ANALYSIS.—Since 8 dollars will buy one barrel, 56 dollars will buy as many barrels as 8 is contained times in 56, which is 7 times. Therefore 7 barrels of flour, at 8 dollars each, can be bought for 56 dollars.

3. If a man can dig 6 rods of ditch in a day, how many days will it take him to dig 96 rods?

4. A farmer bought 49 sheep for 196 dollars; what did they cost a piece?

42. **Division** is the process of finding how many times one number is contained in another.

43. The **Dividend** is the number to be divided.

44. The **Divisor** is the number divided by.

45. The **Quotient** is the result obtained by the process of division, and shows how many times the divisor is contained in the dividend.

NOTE.—1. When the dividend does not contain the divisor an exact number of times, the part of the dividend left is called the *remainder*, and it must be less than the divisor.

2. As the remainder is always part of the dividend, it is always of the same name or kind.

3. When there is no remainder the division is said to be *complete*.

46. The sign, ÷, placed between two numbers, denotes division, and shows that the number on the *left* is to be divided by the number on the *right*. Thus, 39 ÷ 3, is read 39 *divided by* 3.

Division is often indicated by writing the dividend *above* and the divisor *below* a short horizontal line. Thus, $\frac{39}{3}$

CASE I.

47. *When the divisor does not exceed* 12.

EXAMPLE 1.—How many times is 3 contained in 936?

OPERATION.

	Dividend.
Divisor	3)936
Quotient	312

ANALYSIS.—After writing the divisor on the left of the dividend, with a line between them, we begin at the left hand and say: 3 is contained in 9 hundreds, 3 hundreds times, and write 3 in hundreds' place in the quotient:

then 3 is contained in 3 tens 1 ten times, and write 1 in tens' place in the quotient; then 3 is contained in 6 units 2 units times; and writing the 2 in units' place in the quotient, we have the entire quotient, 312.

2. How many times is 4 contained in 1684?

OPERATION.

4)1684
———
421

ANALYSIS.—As we cannot divide 1 thousand by 4, we take the 1 thousand and the 6 hundreds together, and say, 4 is contained in 16 hundreds 4 hundreds times, which we write in hundreds' place in the quotient; then 4 is contained in 8 tens 2 tens times, which we write in the tens' place in the quotient; and 4 is contained in 4 units 1 unit time, which we write in the units' place in the quotient, and we have the entire quotient, 421.

3. How many times is 7 contained in 2835?

OPERATION.

7)2835
———
405

ANALYSIS.—Beginning as in the last example, we say, 7 is contained in 28 hundreds 4 hundreds times, which we write in the hundreds' place in the quotient; then, 7 is contained in 3 tens no times, and we write a cipher in the tens' place in the quotient; and taking the 3 tens and 5 units together, 7 is contained in 35 units 5 units times, which we write in the units' place in the quotient, and we have the entire quotient, 405.

4. How many times is 8 contained in 987?

OPERATION.

8)987
———
123 3 Rem.
or
$123\frac{3}{8}$

ANALYSIS.—Here 8 is contained in 9 hundreds 1 hundred times, and 1 hundred, or 10 tens, over, which, united to the 8 tens, make 18 tens; 8 in 18 tens, 2 tens times and 2 tens, or 20 units, over, which, united to the 7 units, make 27 units; 8 in 27 units 3 units times and 3 units over. The 3 which is left after performing the division, should be divided by 8; but the method of doing so cannot be explained until we reach *fractions;* so we merely indicate the division by placing the divisor under the dividend, thus, $\frac{3}{8}$. (**46**). The entire quotient is written $123\frac{3}{8}$, which may be read, one hundred and twenty-three and *three-eighths*, or one hundred and twenty-three and a *remainder of three.*

From the foregoing examples and illustrations, we deduce the following

RULE. I. Write the divisor at the left of the dividend, with a line between them.

II. Beginning at the left hand, find how many times the divisor is contained in the fewest number of figures of the dividend that will contain it, and write the result under the dividend.

III. If there be a remainder after dividing any figure, regard it as prefixed to the figure of the next lower order in the dividend, and divide as before.

IV. Should any figure or part of the dividend be less than the divisor, write a cipher in the quotient, and prefix the number to the figure of the next lower order in the dividend, and divide as before.

V. If there be a remainder after dividing the last figure, place it over the divisor at the right hand of the quotient.

Mental Exercises.

1. If 4 casks of lime cost 12 dollars, what is the cost of 1 cask?

2. If a man perform a certain piece of work in 30 days, how long will it take 5 men to do the same? How long will it take 6 men? How long will it take 7 men?

3. If 24 pounds of tea can be purchased for 12 dollars, how much can be bought for 1 dollar? How much for 9 dollars? How much for 5 dollars?

4. Gave 96 cents for 6 pounds of raisins; what cost 1 pound? What cost 7 pounds?

5. A man gave 15 dollars for 3 barrels of apples; what was the cost of each barrel? What would 5 barrels cost at the same rate?

Exercises for the Slate.

SECTION I.

(1) $42240 \div 2, 4, 6, 8, 10, 11$	(5) $30888 \div 9, 3, 8$
(2) $14784 \div 3, 7, 11, 2, 4, 8$	(6) $13608 \div 7, 3, 9$
(3) $76032 \div 4, 3, 2, 8, 9, 11$	(7) $34668 \div 6, 9, 3$
(4) $120960 \div 5, 7, 6, 4, 8$	(8) $363285 \div 5, 9, 3$

SHOW THAT

(9) $369 \div 3 = 246 \div 3 + 123 \div 3$

(10) $1035 \div 5 = 690 \div 5 + 345 \div 5$

(11) $1368 \div 4 = 912 \div 4 + 456 \div 4$

(12) $1701 \div 7 = 1134 \div 7 + 567 \div 7$

(13) $7866 \div 9 = 3231 \div 9 + 4635 \div 9$

SECTION II.

		Quotients.			Quotients
(1)	42544830 ÷ 6 =	7090805	(6)	49368768 ÷ 6 =	8228128
(2)	14284263 ÷ 7 =	2040609	(7)	28949076 ÷ 12 =	2412423
(3)	24486456 ÷ 8 =	3060807	(8)	59987688 ÷ 12 =	4998974
(4)	67879284 ÷ 6 =	11313214	(9)	23935734 ÷ 6 =	3989289
(5)	78485617 ÷ 7 =	11212231	(10)	98765711 ÷ 11 =	8978701

		Quotients.	Rem.
(11)	7341568 ÷ 7		
	3179632 ÷ 5		
	19038716 ÷ 8		
	84201763 ÷ 9		
	2947691 ÷ 12		
	42084796 ÷ 6		

Sum of Quotients and Remainders 20680083—28.

CASE II.

48. *When the divisor is a composite number.*

EXAMPLE 1.—If 5376 dollars be divided equally among 42 men, how many dollars will each receive?

OPERATION.

```
6)5376
  ————
  7)896
    ———
    128 Ans.
```

ANALYSIS.—If 5376 dollars be divided equally among 42 men, each man will receive as many dollars as 42 is contained in 5376 dollars. 42 may be resolved into the factors 6 and 7; and we may suppose the 42 men divided into six groups of 7 men each; dividing the 5376 by 6, the number of groups, we have 896, the number of dollars to be given to each group; and dividing 896 by 7, the number of men in each group, we have 128, the number of dollars that each man will receive. Hence,

RULE. Divide the dividend by one of the factors, and the quotient thus obtained by another, and so on if there be more than two factors, until every factor has been made a divisor. The last quotient will be the quotient required.

SECTION III.

					Quotients.
1.	Divide	985768545	by	15 = 3 × 5	65717903.
2.	Divide	687698464	by	16 = 4 × 4	42981154.
3.	Divide	931684770	by	45 = 5 × 9	20704106.
4.	Divide	945328608	by	56 = 8 × 7	16880868.
5.	Divide	3948767388	by	108 = 3 × 4 × 9	36562661.
6.	Divide	3176823672	by	132 = 12 × 11	24066846.

40. *To find the true remainder.*

EXAMPLE 2.—Divide 1143 by 64, using the factors 2, 8, and 4, and find the true remainder.

OPERATION.

2)1143		
8)571		1 rem.
4)71	$3 \times 2 =$	6
17	$3 \times 8 \times 2 =$	48
		55 true rem.

ANALYSIS.—Dividing 1143 by 2 we get a remainder of 1 undivided, which being a part of the given dividend must also be a part of the true remainder.—And in dividing the first quotient by 8, we get a remainder 3, which we multiply by 2, the first divisor, to bring it to the same name, or units, as the first remainder, and in dividing by 4, we have a remainder of 3, which we multiply by 8 and 2, the preceding divisors, in order to bring it also to the same name as the first remainder. Adding the three partial remainders, we obtain 55, the true remainder. Hence the

RULE. I. Multiply each partial remainder, except the first, by all the preceding divisors.

II. Add the several products with the first remainder, and the sum will be the true remainder.

NOTE.—For other methods see Advanced Arithmetic.

SECTION IV.

1. 234567 ÷ 18	6. 751113 ÷ 63	11. 23456781 ÷ 216
2. 345672 ÷ 27	7. 804024 ÷ 72	12. 83456712 ÷ 225
3. 427311 ÷ 36	8. 887625 ÷ 81	13. 40107645 ÷ 432
4. 453672 ÷ 45	9. 999999 ÷ 99	14. 57763323 ÷ 441
5. 672345 ÷ 54	10. 723456 ÷ 108	15. 68960286 ÷ 504

SECTION V.

1.	958768461 ÷ 27	Ans.	35509943.
2.	726894784 ÷ 32	"	22715462.
3.	729368465 ÷ 35	"	20839099.
4.	675487368 ÷ 36	"	18763538.
5.	945328608 ÷ 56	"	16880868.
6.	1796842688 ÷ 64	"	28075667.
7.	897684192 ÷ 72	"	12467836.
8.	910364312 ÷ 88	"	10345049.
9.	3948767388 ÷ 108	"	36562661.
10.	3176823672 ÷ 132	"	24066846.

CASE III.

50. *To divide by a number consisting of several figures.*

NOTE.—To illustrate the method of operation more clearly, we will take an example usually performed by Short Division.

1. How many times is 6 contained in 564.

OPERATION.

```
6)564(94
  54
  ——
   24
   24
   ——
```

ANALYSIS.—As 6 is not contained in 5 hundreds, we take 5 and 6 as one number, and consider how many times 6 is contained in this *partial* dividend, 56 tens, and find that it is contained 9 tens times, and a remainder. To find this remainder, we multiply the divisor, 6, by the quotient figure, 9 tens, and subtract the product, 54 tens, from the partial dividend, 56 tens, and there remain 2 tens. To this remainder we bring down the 4 units, and consider the 24 units the *second* partial dividend. Then, 6 is contained in 24 units 4 units times. Multiplying and subtracting as before, we find that nothing remains, and we have for the entire quotient, 94.

2. How many times is 23 contained in 4807 ?

OPERATION.

```
Divisor. Dividend. Quotient.
  23  )  4807  (  209
         46
         ——
          207
          207
          ——
```

ANALYSIS.—We first find how many times 23 is contained in 48, the least number of figures that will contain 23, and place the result in the quotient on the right of the dividend. We then multiply the divisor, 23, by the quotient figure, 2, and subtract the product, 46, from the part of the dividend used, and to the remainder bring down the next figure of the dividend, which is 0, making 20, for the second partial dividend. Then, since 23 is contained in 20 no times, we place a cipher in the quotient, and bring down the next figure of the dividend, making a third partial dividend, 207; 23 is contained in 207, 9 times: multiplying and subtracting as before, nothing remains, and we have for the entire quotient, 209.

NOTES.—1. When the process of dividing is performed mentally, and the results only are written, as in Case I, the operation is termed *Short Division*.

2. When the whole process of division is written, the operation is termed *Long Division*.

From the preceding illustrations we derive the following general

RULE. I. Write the divisor at the left of the dividend, as in Short Division.

II. Divide the least number of the left hand figures in the dividend that will contain the divisor one or more times, and place the quotient at the right of the dividend, with a line between them.

III. Multiply the divisor by this quotient figure, subtract the product from the partial dividend used, and to the remainder bring down the next figure of the dividend.

IV. Divide as before, until all the figures of the dividend have been brought down and divided.

V. If any partial dividend will not contain the divisor, place a cipher in the quotient, and bring down the next figure of the dividend, and divide as before.

VI. If there be a remainder after dividing all the figures of the dividend, it must be written in the quotient, with the divisor underneath.

NOTE.—1. If any remainder be *equal to*, or *greater* than the divisor, the quotient figure is too *small*, and must be increased.

2. If the product of the divisor by the quotient figure be *greater* than the partial dividend, the quotient figure is *too large*, and must be diminished.

SECTION VI.

(1) 79865379÷702	(6) 53146827 ÷459	(11) 709005474÷882
(2) 81136863÷801	(7) 61327548 ÷558	(12) 407049570÷918
(3) 90909963÷117	(8) 128713536÷567	(13) 981234567÷891
(4) 23659245÷126	(9) 123456789÷576	(14) 900664200÷9099
(5) 37018764÷135	(10) 987654321÷585	(15) 111777111÷9009

SECTION VII.

1. Divide 5560804464 by 7346. Ans. 756984.
2. Divide 1747071255 by 6483. Ans. 269485.
3. Divide 8287864532 by 8594. Ans. 964378.
4. Divide 35365114332 by 93846. Ans. 376842.
5. Divide 520090972776 by 654321. Ans. 794856.
6. Divide 7428927415293 by 8496427. Ans. 874359.
7. Divide 936864880704 by 987654. Ans. 948576.

8 The number of post offices in the United States in 1853 was 22320, and the revenue of this department was 5937120 dollars; what was the average revenue of each office? Ans. 266 dollars.

9. A bag containing three hundred and twenty-four nuts was divided among nine boys; how many did each boy get? Ans. 36.

10. Find the 17th part of 5508. Ans. 324.

11. How many miles an hour does a train go which travels 1692 miles in 47 hours? Ans. 36.

12. A gentleman left £5000. By his will he directed that after paying his debts, amounting to £275, the rest

should be divided equally among his seven children; what was the share of each? Ans. £675.

13. The product of two numbers is 31383450, and one of the numbers is 4050; what is the other number? Ans. 7749.

CASE IV.

51. *To divide by* 10, 100, 1000, &c.

EXAMPLE 1.—Divide 486 acres of land equally among 10 men; how many acres will each have?

OPERATION.

1|0)48|6

48 6 rem.

ANALYSIS.—According to the decimal system of notation if we remove a figure one place toward the left by annexing a cipher, its value is increased ten fold, or is *multiplied* by 10, so on the contrary, by cutting off, or taking away the right hand figure of a number, each of the figures is removed one place toward the right, and consequently reduced to one-tenth its former value, or divided by 10.

For similar reasons, if we cut off *two* figures we divide by 100, if *three*, we divide by 1000, and so on. Hence the

RULE. From the right hand of the dividend cut off as many figures as there are ciphers in the divisor. Under the figures so cut off, place the divisor, and the whole will form the quotient.

52. *To divide by a number having ciphers on the right hand.*

EXAMPLE 1.—Divide 587618 by 400.

OPERATION.

4|00)5876|18

1469 18 rem.

ANALYSIS.—In this example we resolve 400 into the factors, 4 and 100, and divide first by 100, by cutting off the two right hand figures of the dividend, (**51**) and we have a quotient of 5876, and a remainder of 18. We next divide by 4, and obtain 1469 for a quotient; and the entire quotient is $1469\frac{18}{400}$.

53. When there is a remainder after dividing by the significant figures, it must be prefixed to the figures cut off from the dividend to give the true remainder.

SECTION VIII.

1. Divide 48600 by 100. Ans. 486.
2. Divide 59673 by 1000. Ans. 59 rem. 673 or $59\frac{673}{1000}$.
3. Divide 34716 by 900. Ans. 38 rem. 516 or $38\frac{516}{900}$.
4. Divide 178930 by 10. Ans. 17893.

5. Divide 47321046 by 45000. Ans. 1051, rem. 26046
Or $1051\frac{26046}{45000}$.

6. Divide 1047634 by 2400. Ans. 436, rem. 1234
Or $436\frac{1234}{2400}$.

7. The sum of 40000 dollars is paid to 1600 men; what does each man receive? Ans. 25 dollars.

8. The circumference of the earth at the equator is 24898 miles. How many hours would a train of cars require to travel that distance, going at the rate of 60 miles an hour?
Ans. $414\frac{58}{60}$.

SECTION VIII.

To *one*, annex as many ciphers as you please. From this subtract any number. To the two numbers thus formed, *prefix* two figures whose sum is less than the proposed divisor by *one*, then divide by the proposed divisor.

EXAMPLE 1.—To 1, annex 5 ciphers. Thus, 100000
From this subtract any number (say) 54321 *(a)*

45679 *(b)*

Take any divisor, as 9. To *(a)* and *(b)* prefix two figures whose sum = 9 less 1, *i. e.* to 8. Say 6 and 2, then—

```
9)6,54321        9)2,45679
  -------          -------
  72702 3/9        27297 6/9
```

Answers $\begin{cases} 72702\frac{3}{9} & (a) \\ 27297\frac{6}{9} & (b) \end{cases}$

Sum of do. 100000

For Long Division take, say 54. Prefix as before.

```
54)27,54321(51005 51/54      54)26,45679(48994 3/54
   27 0····                     21 6····
   ----                         ----
     54                          485
     54                          432
     ----                        ----
      321                         536
      270                         486
      ---                         ----
      51/54                        507
                                   486
                                   ----
                                    219
                                    216
                                    ---
                                    3/54
```

ANSWERS.
(a) $51005\frac{51}{54}$
(b) $48994\frac{3}{54}$

Sum of do. 100000

MULTIPLICATION AND DIVISION BY FRACTIONAL NUMBERS.

EXAMPLE 1.—Multiply 1483 by $123\frac{5}{8}$.

OPERATION.

$$\begin{array}{r} 1483 \\ 123\frac{5}{8} \\ \hline 4449 \\ 2966 \\ 1483 \\ 926\frac{7}{8} \\ \hline 183335\frac{7}{8} \end{array}$$

ANALYSIS.—Here we multiply 1483 by 123 in the usual way; but before adding the partial products we find the 5 eighths of 1483, namely $926\frac{7}{8}$, and write it under the partial products, as in addition, then adding the four lines we obtain the required product.

We multiply by $\frac{5}{8}$ (or any other fraction) by multiplying the given number by the upper number of the given fraction and dividing the product by the lower. Thus, 1483×5 (the upper figure) $= 7415 \div 8$ (the lower figure) $= 926\frac{7}{8}$.

EXAMPLE 2.—Divide 1234 by $4\frac{3}{4}$.

OPERATION.

$$\begin{array}{l} 4\frac{3}{4})1234 \\ \underline{4}\quad\ \ \underline{\quad 4} \\ 19\ )4936(259\frac{15}{19} \\ \quad\ \ \underline{38} \\ \quad\ \ 113 \\ \quad\ \ \ \underline{95} \\ \quad\ \ \ 186 \\ \quad\ \ \ \underline{171} \\ \quad\ \ \ \ \frac{15}{19} \end{array}$$

ANALYSIS.—We first bring both divisor and dividend to the same name as the given fraction—that is (in this instance) to fourths, then proceed as in division.

Exercises for the Slate.

(1)	$18947632 \times 5\frac{1}{2}$	Ans. 104211976
(2)	$46738479 \times 6\frac{1}{2}$	$303800113\frac{1}{2}$
(3)	$94327865 \times 30\frac{1}{4}$	$2853417916\frac{1}{4}$
(4)	$29768342 \times 10\frac{2}{3}$	$317528981\frac{1}{3}$
(5)	$29648732 \times 2006\frac{10}{11}$	$59502309784\frac{8}{11}$
(6)	$43796284 \div 6\frac{1}{2}$	$6737889\frac{11}{13}$
(7)	$49625483 \div 30\frac{1}{4}$	$1640511\frac{101}{121}$
(8)	$876587938 \div 148\frac{2}{7}$	$5911479\frac{864}{1038}$

PROMISCUOUS EXERCISES IN THE PRECEDING RULES.

1. One school contains 60 pupils, a second 83, a third 125, a fourth 234, a fifth 672, and a sixth 1003; how many pupils are there in the six schools? Ans. 2177.

2. The Clyde is 100 miles long, the Forth 115, the Thames 215, the Shannon 224, and the Severn 240; what would be the length of a river equal to them all? Ans. 894 miles.

3. What is the difference between 8964 and 14,382? Ans. 5418.

4. Two factors are 57682 and 8493; what is their product? Ans. 489893226.

5. How much less is 7289 than 8723? Ans. 1434.

6. There are 4 chests of drawers; in each chest there are 12 drawers, and in each drawer there are placed 12 dollars; how many dollars are there altogether in the chests? Ans. 576 dollars.

7. Multiply 94836 by 768, and divide the product by 9216. Ans. 7903.

8. From the sum of 189649, 283726, 542893, 248567, 693284 and 256893 subtract 48972, multiply the remainder by 84762, and divide the product by 9418. Ans. 19494360.

9. A man commenced business when 22 years old, and retired at the age of seventy with a fortune of 48768 dollars. Required how much he cleared on an average each year? Ans. 1016 dollars.

10. A wood of 6723 trees is to be thinned by cutting down one tree in nine; how many will be left after this clearing? Ans. 5976.

PRIME NUMBERS.

54. A Prime Number is one that cannot be resolved into two or more integral factors; thus 7, 3, 11, &c., are *prime* because they are not divisible by any number greater than 1, without a remainder.

55. *To find the prime factors of any composite number.*

EXAMPLE 1.—What are the prime factors of 30?

OPERATION.

```
2 | 30
3 | 15
5 |  5
  |  1
```

ANALYSIS.—We divide the given number by 2, the least prime factor; this gives an odd number for the quotient, divisible by the prime factor, 3, and obtain the quotient 5; this being a prime number, the division cannot be carried any further. The divisors and the last quotient, 2, 3 and 5, are all the prime factors of the given number, 30. Hence the proof $2 \times 3 \times 5 \times 1 = 30$.

RULE. Divide the given number by any prime factor; divide the quotient in the same manner, and so continue the division until the quotient is a prime number. The several divisors and the last quotient will be the prime factors required.

Mental Exercises.

1. What are the prime factors of 9, 12, 15, 16 and 18 ?
2. What are the prime factors of 39, 26, 34, 38 and 42 ?
3. What are the prime factors of 65, 85, 95, 105 and 115 ?

Exercises for the Slate.

Find the prime factors of the following numbers and prove he results.

(1) 15	(5) 39	(9) 57	(13) 85	(17) 120	(21) 1492
(2) 18	(6) 42	(10) 69	(14) 91	(18) 144	(22) 8032
(3) 24	(7) 45	(11) 78	(15) 99	(19) 714	(23) 4604
(4) 36	(8) 49	(12) 88	(16) 108	(20) 836	(24) 1728

GREATEST COMMON MEASURE.

56. A **Common Divisor** of two or more numbers is a number that will exactly divide each of them.

57. The **Greatest Common Divisor** of two or more numbers is the greatest number that will exactly divide each of them.

Numbers prime to each other are such as have no common divisor.

NOTE.—A common divisor is called a common measure; and the greatest common divisor, the greatest common measure. The latter is usually indicated by the initial letters G. C. M.

58. *To find the greatest common measure of two numbers.*

EX.—Find the greatest common measure of 105 and 165.

OPERATION.

```
105)165(1
    105
    ———
     60)105(1
         60
         ——
         45)60(1
            45
            ——
            15)45(3
               45
               ——
                0
```

ANALYSIS.—Here we divide the greater number, 165, by the less, 105, and thus obtain a remainder, 60, which we now make a divisor, and 105, the former divisor, the dividend, and so on. When the remainder, 15, is used as a divisor it leaves no remainder, and is therefore the greatest common measure required. Hence,

RULE. I. Divide the greater number by the less.

II. Divide the preceding divisor by the last remainder, and so on till nothing remains. The last divisor will be the greatest common measure.

59. *To find the greatest common measure of three or more given numbers.*

RULE. I. Find the greatest common measure of any two of the given numbers, by the last rule.

II. Then, that of the common divisor thus obtained and of another of the given numbers, and so on through all the given numbers.

III. The last common divisor found will be the greatest common measure of all the given numbers.

Exercises for the Slate.

SECTION I.

Find the greatest common measure of

(1) 12 and 18.	Ans. 6	(6) 1024 and 2240.	Ans. 64
(2) 21 and 28.	7	(7) 1624 and 14500.	116
(3) 39 and 52.	13	(8) 714 and 1176.	42
(4) 42 and 77.	7	(9) 21671 and 22111.	prime
(5) 28 and 126.	14	(10) 11256 and 19899.	201

11. What is the greatest common divisor of 72, 120, 240, and 384? Ans. 24.

12. What is the greatest common measure of 300, 525, 225, and 375? Ans. 75.

EXAMPLE 2.—Find the greatest common measure of 42, 63, and 105.

OPERATION.

$42 = 2 \times 3 \times 7$ prime factors.
$63 = 3 \times 3 \times 7$ " "
$105 = 3 \times 5 \times 7$ " "

The factors common to the three given numbers are 3 and 7. Therefore $3 \times 7 = 21$, the greatest common measure. Hence,

RULE. I. Resolve each number into its prime factors.

II. Select those which are common to all the numbers, and their product will be their greatest common measure.

SECTION II.

Find the greatest common measure of

(1) 12, 36, 60 and 72.	Ans. 12	(5) 200, 625, and 150.	Ans. 25
(2) 18, 24, 30, 36 and 42.	6	(6) 252, 630, 1134 and 1386.	126
(3) 36, 126, 72, 216.	18	(7) 28, 140 and 280.	28
(4) 32, 80 and 256.	16	(8) 468 and 1184.	4

LEAST COMMON MULTIPLE.

60. A **Multiple** is a number exactly divisible by a given number; thus 16 is a multiple of 4.

61. A **Common Multiple** is a number exactly divisible by two or more given numbers; thus, 16 is a common multiple of 2, 4, and 8.

62. The **Least Common Multiple** is the least number exactly divisible by two or more given numbers; thus 24 is the least common multiple of 2, 4, 6, and 8.

63. *To find the least Common Multiple of two or more given numbers.*

EXAMPLE 1.—Find the least common multiple of 12, 30, 42 and 66.

OPERATION.

$12 = 3 \times 2 \times 2$ prime factors.
$30 = 3 \times 2 \times 5$ "
$42 = 3 \times 2 \times 7$ "
$66 = 3 \times 2 \times 11$ "

$3 \times 2 \times 11 \times 7 \times 5 \times 2 = 4260$, Ans.

ANALYSIS.—The number cannot be less than 66, since it must contain 66; hence it must contain the factors of 66, viz.,

$$3 \times 2 \times 11$$

We have all the prime factors of 66, and also the prime factors of 42, except the factor 7. Annexing 7 to the series of factors,

$$3 \times 2 \times 11 \times 7$$

and we have all the prime factors of 66 and 42, and also all the factors of 60, except the factor 5. Annexing 5 to the series of factors,

$$3 \times 2 \times 11 \times 7 \times 5$$

and we have all the prime factors of 66, 42, and 60, and also all the factors of 12 except the factor 2. Annexing 2 to the series of factors,

$$3 \times 2 \times 11 \times 7 \times 5 \times 2$$

and we have all the prime factors of each of the given numbers; and hence the product of the series of factors is a common multiple of the given numbers.

As no factor of the series can be omitted without omitting a factor of one of the given numbers, the product of the series is the least common multiple of the given numbers.

From this illustration we deduce the following

RULE. I. Resolve the given numbers into their prime factors.

II. **Take all the prime factors of the largest number, and such prime factors of the other numbers as are not found in the largest number, and their product will be the least common multiple.**

NOTE.—For other methods see Advanced Arithmetic.

Find the least common multiple of the following numbers.

1. 7, 35 and 98. Ans. 490
2. 4, 9, 6 and 8. 72.
3. 8, 15, 77 and 385. 9240.
4. 12, 15, 42 and 60. 420.
5. 21, 35 and 42. 210.
6. 4, 16, 20, 48, 60 and 72. 720.
7. 5, 10, 15, 20, 25, 30, 35 and 40. 4200.
8. 3, 6, 9, 12, 48, 21, 24 and 16. 1008.
9. 15, 12, 128, 30, 16, 4, 320 and 96. 1920
10. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30 and 32. Ans. 1441440.
11. What is the smallest sum of money for which I could purchase an exact number of books, at 5 dollars, or 3 dollars, or 4 dollars, or 6 dollars each? Ans. 60 dollars.

— —

DECIMALS.

64. Decimal Fractions are the decimal divisions of a unit; thus a unit is divided into ten equal parts called *tenths;* each of these tenths is divided into ten other equal parts called *hundredths;* and so on. Since the denominators of decimal fractions increase and decrease by the scale of 10, the same as simple numbers, in writing decimals the denominators are generally omitted.

65. In simple numbers the unit 1, is the starting point of notation and numeration; and so also is it in decimals.

66. The **Decimal Point** is a period, (.) which must always be placed before the left hand figure of the decimal. Thus,

$\frac{6}{10}$ is expressed .6

$\frac{567}{1000}$ " .567

67. The names of the different *orders of decimals*, or places below units, may be easily learned from the following

Decimal Table.

			&c.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.
0	0	0	0	0	0	0	0	0	.	0	0	0	0	0	0	0	0	0
										1st place,	2d place,	3d place,	4th place,	5th place,	6th place,	7th place,	8th place,	9th place,

By examining this table we see that

Tenths are expressed	by	one figure.
Hundredths	"	two figures.
Thousandths	"	three figures.

68. Every cipher on the left hand of a decimal reduces it to one-tenth its previous value. Thus, .5 is 5 tenths, .05 is 5 hundredths, and .005 is 5 thousandths.

Ciphers on the right do not alter the value, for .5, .50, .500 are the same as $\frac{5}{10}$, $\frac{50}{100}$, $\frac{500}{1000}$, and these are all equal.

NOTATION AND NUMERATION OF DECIMALS.

69. *Rule for decimal notation.*

I. Write the decimals as a whole number, placing ciphers where necessary to give each significant figure its true local value.

II. Place the decimal point before the first figure.

70. *Rule for decimal numeration.*

RULE. I. Numerate from the decimal point, to determine the denominator.

II. Numerate towards the decimal point, to determine the numerator.

III. Read the decimal as a whole number, giving it the mean or denomination of the right hand figure.

Exercises for the Slate.

1. Write 265 ten thousandths.
2. Write six hundred and thirteen thousandths.
3. Write 365 thousands, and 4 billionths.
4. Write seven hundred thousandths.
5. Write one hundred, and 2 tenths.
6. Read the following numbers:

1.265	4.0005	6.0007
3.898	17.2006	1267.9876543
.5967	119.3200	3.0000678
46.7325	.5000	123.45607890

ADDITION OF DECIMALS.

71. EXAMPLE 1.—Add 3 tenths, 45 hundredths, 16 tenths, and 365 thousandths.

OPERATION.

```
  .3
  .45
 1.6
  .365
 -----
 2.715
```

ANALYSIS.—As in simple numbers, we write the numbers so that units shall stand under units, tenths under tenths, hundredths under hundredths, &c. This brings the decimal points directly under each other. Commencing at the right hand we add each column, and carry as in whole numbers, and in the result we place a point between the units and tenths, or directly under the decimal point in the numbers added. Hence the

RULE. I. Write the numbers so that the decimal points shall stand directly under each other.

II. Add as in whole numbers, and place the decimal point, in the result, directly under the points in the numbers added.

Mental Exercises.

1. Add .6 and .06; 10 and .01; 3.6 and 3.007; .8 and .9.
2. Add 6 hundredths and 56 thousandths; .06 and .056.
3. Add 20 cents and 156 cents; .20 and 1.56.
4. Add 256 dollars and 3 dollars and 25 cents; 256 + 3 + .25.

Exercises for the Slate.

SECTION I.

(1) 27.655 + 71.784 + 98.687 + 84.769.
(2) 219.373 + 376.458 + 843.847 + 591.738 + 456.153.
(3) 26.3756 + 74.5673 + 56.8948 + 74.7355 + 53.1052.
(4) 254.172 + 888.627 + 568.296 + 756 939 + 531.704.
(5) 214.735 + 607.434 + 669.758 + 496.376 + 730.242.

SECTION II.

1. Add 25.7, 8.389, 23.056. Ans. 57.145.
2. Add 36.258, 2.0675, 382.45. Ans. 420.7755.
3. Add 32.764, 5.78, 16.0037 and 49.3046. Ans. 103.8523.
4. Add 1152.01, 14.11018, 152348.21, 9.000083. Ans. 153523.330263.
5. Add 37.03, 0.521, .9, 1000, 4000.0004. Ans. 5038.4514.
6. What is the sum of twenty-six, and twenty-six hundredths; seven tenths; six, and eighty-three thousandths; four, and four thousandths? Ans. 37.047.
7. How many yards in three pieces of cloth, the first piece containing 18.375 yards, the second piece 41.625 yards, and the third piece 35.5 yards? Ans. 95.5 yards.

SUBTRACTION OF DECIMALS.

72. Example 1.—From 31.63 take 27.85.

OPERATION.

```
 31.63
 27.85
 -----
  3.78
```

Ex. 2.—From 3.8674 take 1.36.

OPERATION.

```
 3.8674
 1.36
 ------
 2.5074
```

Ex. 3.—From 15.36 take 8.1234

OPERATION.

```
 15.36
  8.1234
 -------
  7.2366
```

Analysis.—In each of these three examples, we write the subtrahend under the minuend, placing units under units, tenths under tenths, &c. Commencing at the right hand we subtract as in whole numbers, and in the remainders we place the decimal points directly under those in the numbers above. In the second example the number of decimal places in the minuend is greater than the number in the subtrahend, and in the third example less. In both cases, we reduce both minuend and subtrahend to the same name, or number of decimal places, by annexing ciphers; or we suppose them to be annexed before performing the subtraction.—Hence,

RULE. Place the numbers as in addition, subtract as in simple numbers, and insert the decimal point directly under the points in the given numbers.

Mental Exercises.

1. From five tenths take forty-nine hundredths.
2. From .63 take .496; 2.19 take .63; .5 take .005.
3. From .16 take .006; 12.34 take 2.345; 100 take .001.
4. From one take two hundredths.
5. From 3.10 dollars take 75 cents; 3.10 take .75.

Exercises for the Slate.

SECTION I.

1. From 20.34	take	13.56		5. From 52.0704	take	34.7136
2. From 40.68	"	27.12		6. From 430.2816	"	286.8544
3. From 16.272	"	10.848		7. From 2603.52	"	1735.68
4. From 6.5088	"	4.3392		8. From 983.9607	"	655.9738

SECTION II.

Find the value of—

(1) 111.1116—22.22222.	Ans. 88.88938	(5) 21.004—.75	Ans. 20.254
(2) 279.00906—117.916.	161.09306	(6) 714.0—.916	713.084
(3) 8.135—2.6875.	5.4475	(7) 2—.298	1.702
(4) 627.4—91.7469	535.6531	(8) 1000—.001	999.999

MULTIPLICATION OF DECIMALS.

73. Example.—What is the product of .25 multiplied by .5

OPERATION.
.25
.5
.125

Analysis.—We perform the multiplication the same as in whole numbers. Since the multiplicand is 25 hundredths, and the multiplier 5 tenths, and hundredths multiplied by tenths give thousandths, and thousandths being expressed by three figures, we must have three places of decimals in the product. Hence we see the product contains as many decimal places as are contained in both multiplicand and multiplier. Hence,

RULE. Multiply as in whole numbers, and from the right hand of the product point off as many figures for decimals as there are decimal places in both factors.

Note 1.—If there are not as many figures in the product as there are decimals in both factors, supply the deficiency by *prefixing* ciphers.

2.—To multiply by 10, 100, 1000, &c., remove the decimal point as many places to the right as there are ciphers on the right of the multiplier.

Mental Exercises.

1. If a man can reap .96 of an acre in a day, how much can he reap in .5 ofa day?
2. If 1 pound of coffee cost .3 of a dollar, what will 4 pounds cost?
3. Add 3.6 + .26 + .006 + 3.006, and multiply the product by .8
4. From 3.606 take 1.4, and multiply the result by .09
5. If 1 ton of hay cost 8.75 dollars, what will .25 of a ton cost?

Exercises for the Slate.

SECTION I.

Multiply and add together the products of—

(1) 1234.56789 by 78.91 and 21.09	(6) by 550.8 and 449.2
(2) 345.789612 by 35.79 and 64.21	(7) by 900.9 and 99.1
(3) 406.783089 by 60.09 and 39.91	(8) by 428.6 and 571.4
(4) 2492.67339 by 42.82 and 57.18	(9) by 624.8 and 375.2
(5) 5063.48001 by .99 and 99.01	(10) by 99.73 and .27

SECTION II.

Find the product of—

(1) .132 × .241	Ans. .031812	(6) .0006 × .00012	Ans. .000000072
(2) .23 × .009	.00207	(7) 8.0004 × .004	.0320016
(3) 21.716 × 2.06	44.73496	(8) 164.023 × 12.88	2112.61624
(4) 11.111 × 9.7116	107.9055876	(9) 178.006 × 100.001	17800.778006
(5) .2 × .7 × .06 × .004 × .1	.00000336	(10) 43.1 × .6 × 100. × .01	25.86

11. Multiply four hundred, and four thousandths by thirty and three hundredths. Ans. 12012.12012.

12. If a cord of wood be worth 2.37 bushels of wheat, how many bushels of wheat must be given for 9.58 cords of wood? Ans. 22.7046 bushels.

DIVISION OF DECIMALS.

74. EXAMPLE.—What is the quotient of .156 divided by .6

OPERATION.
.6).156
——
Ans. .26

ANALYSIS.—We perform the division as in whole numbers. Since the dividend, which is the product of the divisor and quotient, contains three places, and the divisor contains one place, the quotient must contain two places of decimals for, 2 + 1 = 3, or 3 — 1 = 2, (**73.**) Hence,

RULE. Divide as in whole numbers, and from the right hand of the quotient point off as many places for decimals, as the decimal places in the dividend exceed those of the divisor.

NOTE 1.—The dividend must always contain at least as many places of decimals as the divisor, before commencing the division.

2.—If the number of figures in the quotient be less than the excess of the decimal places in the dividend over those of the divisor, the deficiency must be supplied by *prefixing* ciphers.

3.—To divide by 10, 100, 1000, &c., remove the decimal point as many places to the left as there are ciphers on the right hand of the divisor.

Mental Exercises.

1. How many bushels of oats at .2 of a dollar a bushel, can be bought for .84 of a dollar?
2. If 15 pounds of coffee cost 4.50 dollars, what cost 1 pound?
3. If a team can plough .75 of an acre in .5 of a day, how much will it plough in one day?
4. How many boxes will be required to pack 49.5 pounds of butter, if you put 5.5 pounds in each?
5. If a man can walk 16.5 miles in a day, how long will it take him to walk 36.30 miles?

Exercises for the Slate.

SECTION I.

Find the value of—

(1) 3448116.1269 ÷ .2349	(7) 218.05605 ÷ 17685
(2) 5096.49732 ÷ 3.726	(8) 7513.866909 ÷ 146.7
(3) 50964.9732 ÷ 1367.82	(9) 75138.66909 ÷ 5.121927
(4) 2.1805605 ÷ 1233	(10) 2568.047328 ÷ 55.44
(5) .007513866909 ÷ .001467	(11) .000292572 ÷ .001 ÷ .004 ÷ .9
(6) 75.13866909 ÷ 5.121927	(12) 29.2572 ÷ .36

SECTION II.

What is the quotient of—

(1) 46.84 ÷ 7.9	Ans. 5.9291 +	(6) 4. ÷ .00001	Ans. 400000
(2) 67234 ÷ .85	79098.8235 +	(7) 2.39015 ÷ .007	341.45
(3) 60.0001 ÷ 1.01	59.4060 +	(8) 785.4 ÷ 1000	.7854
(4) 0.00006 ÷ .003	0.02	(9) 3.6 ÷ .00006	60000
(5) 6541.234567 ÷ 21	311.487360 +	(10) .8 ÷ 476.3	.001679 +

11. If 25 men build 154.125 rods of fence in a day, how many does each man build? Ans. 6.165 rods.

12. How many coats can be made from 16.2 yards of cloth, allowing 2.7 yards for each coat? Ans. 6 coats

REDUCTION.

75. A **Concrete Number** is a number of but one name, or denomination; thus, 5 pounds, 27 bushels, 72 dollars, are concrete numbers.

76. A **Compound Number** is a concrete number of two or more denominations; thus, 5 dollars 23 cents, 14 bushels 3 pecks, 9 days 7 hours, are compound numbers.

77. Reduction is the process of changing a number from one denomination to another without altering its value. Reduction is of two kinds, Descending and Ascending.

78. Reduction Descending is changing a number of one denomination to another denomination of *less unit value;* thus 1 dollar = 10 dimes = 100 cents = 1000 mills.

79. Reduction Ascending is changing a number of one denomination to another denomination of *greater unit value;* thus 1000 mills = 100 cents = 10 dimes = 1 dollar.

CURRENCY.

80. Currency is coin, bank bills, &c., in circulation as a medium of trade.

ENGLISH OR STERLING MONEY.

2 Farthings	make	1 Half-penny.		
2 Half-pence	"	1 Penny,	marked	*d.*
12 Pence	"	1 Shilling,	"	*s.*
20 Shillings	"	1 Pound,	"	£.

In Prince Edward Island, Newfoundland, and Jamaica accounts are kept in pounds, shillings, and pence.

CASE I.

81. *To perform Reduction descending.*

EXAMPLE.—Reduce £23 16s. $7\frac{1}{4}$d. to farthings.

OPERATION.

£23	16	$7\frac{1}{4}$
20		
476		
12		
5719		
4		
22877		

ANALYSIS.—Since in £1 there are 20s., in £23 there are 20s. $\times$ 23 $=$ 460s., and 16s. in the given number added, make 476s. in £23 16s. Since in 1s. there are 12d., in 476s. there are 12d. $\times$ 476 $=$ 5712d, and 7d. in the given number added make 5719d. in £23 16s. 7d. Since there are 4 farthings in 1d., in 5719d. there are 4 far. $\times$ 5719 $=$ 22876 far., and 1 far in the given number added makes 22877 far. in £23 16s. $7\frac{1}{4}$d.

NOTE.—When two numbers are to be multiplied together, it is a matter of indifference, so far as the product is concerned, which of them is taken as the multiplicand or multiplier. For convenience we multiply £23 by 20 and call the product shillings, and so with the pence, &c.

Hence the following general

RULE. I. Multiply the highest denomination of the given number by that number in the table which will reduce it to the next lower denomination, and add to the product the given number, if any, of that lower denomination.

II. Proceed in the same manner with the results obtained in each lower denomination, until the reduction is brought to the denomination required.

CASE II.

82. *To perform Reduction ascending.*

EXAMPLE.—Reduce 22877 farthings to pounds.

OPERATION.

4)22877

12)5719d. + 1 far.

2|0)47|6s. + 7d.

£23 16s.

Ans. £23 16s. $7\frac{1}{4}$d.

ANALYSIS.—We first divide the 22877 far. by 4, because there are one-fourth as many pence as farthings, and we find that 22877 far. $=$ 5719d. + 1 far. We next divide 5719d. by 12, because there are one-twelfth as many shillings as pence, and we find that 5719d. $=$ 476s. + 7d. Lastly, we divide the 476s. by 20, because there are one-twentieth as many pounds as shillings, and we find that 476s. $=$ £23 + 16s. The last quotient with the several remainders annexed in the order of the succeeding denominations gives the answer £23 16s. $7\frac{1}{4}$d.—Hence the following general

RULE. I. Divide the given number by that number in the table which will reduce it to the next higher denomination.

II. Divide the quotient by the next higher number in the table; and so proceed to the highest denomination required. The last quotient, with the several remainders annexed in a reversed order, will be the answer.

Mental Exercises.

1. How many farthings are there in 4d.? in 9d.? in $11\frac{1}{2}$d.? in 15d.?
2. How many pence are there in 4s.? in 12s.? in 15s.? in 12s. 6d.?
3. How many pounds, &c., are there in 27s.? in 28s.? in 156s.?
4. How many shillings are there in £6? in £5 7s.? in £6 17s.? in £12 5s.?
5. Five yards of cloth cost £1 2s. 6d.; what was the cost of one yard, in pence?
6. Reduce 960 farthings to pounds. In 690s. how many pounds?
7. What cost 85 pairs of gloves at 7 pence per pair?

Exercises for the Slate.

SECTION I.

Reduce to Farthings.

	£	S.	D.		£	S.	D.		£	S.	D.
(1)	0	1	$8\frac{1}{4}$	(7)	129	3	0	(13)	3974	0	$8\frac{1}{4}$
(2)	1	1	$11\frac{1}{4}$	(8)	103	12	$9\frac{3}{4}$	(14)	1009	15	$5\frac{1}{4}$
(3)	2	7	$7\frac{1}{2}$	(9)	354	10	$10\frac{1}{2}$	(15)	4983	16	$1\frac{1}{2}$
(4)	2	17	$4\frac{1}{2}$	(10)	530	17	$2\frac{1}{4}$	(16)	5993	11	$6\frac{1}{4}$
(5)	2	6	6	(11)	531	2	3	(17)	5221	4	$2\frac{1}{4}$
(6)	28	1	$11\frac{1}{4}$	(12)	531	7	$3\frac{3}{4}$	(18)	5575	15	$0\frac{1}{4}$

19. In £71 13s. $6\frac{1}{2}$d. how many farthings? Ans. 68810.
20. In £295 18s. $3\frac{3}{4}$d. how many farthings. Ans. 284079
21. In 95 guineas, 17s. $9\frac{3}{4}$d., how many farthings? Ans. 96615.
22. Reduce £15 15s. 6d. to sixpences. Ans. 631.
23. Reduce £1[illegible] 14s. 9d. to three pences. Ans. 1259.

SECTION II.

Reduce to Pounds.

(1)	17448	far.	(6)	34904	far.	(11)	21816	half pence.
(2)	43632	"	(7)	78536	"	(12)	21600	"
(3)	138657	"	(8)	198786	"	(13)	99393	"
(4)	156113	"	(9)	302547	"	(14)	224726	pence.
(5)	182289	"	(10)	103753	"	(15)	170663	"

Reduce

(16) 197424 far. to shillings.	(20) 6480 far. to crowns.
(17) 171504 half pence "	(21) 11340 pence "
(18) 756 shillings to guineas.	(22) 2700 " "
(19) 4536 three pences "	(23) 2160 half pence "

24. How many pounds, shillings, &c., are there in 367841 farthings? Ans. £383 3s. $4\frac{1}{4}$d.

25. In 1059120 pence how many sovereigns? Ans. 4413.

26. A farmer, during the year, sold 1367 quarts of milk at 3 pence per quart, what did it all amount to? Ans. £17 1s. 9d.

REDUCTION OF DECIMAL CURRENCY.

83. A **Decimal Currency** is a currency whose denominations increase in a ten-fold ratio, and each denomination is one-tenth the value of the next higher.

The currency of the Dominion of Canada, the United States, France, Barbadoes and some others of the Windward Islands, and Demerara, is decimal.

84. CANADA CURRENCY.

TABLE.

10 Mills (*m*) make 1 Cent, marked *Ct.* or C.
10 Cents " 1 Dime, " *d.*
10 Dimes " 1 Dollar, " $.

NOTE 1.—It is usual in writing dollars and cents, to place the sign ($) of dollars in front of the sum, and a point (.) between the dollars and cents. Thus, fifty-six dollars, four dimes, six cents, and five mills would be written $56.465, or $56.46½, and read 56 dollars and 46½ cents.

2. If the sum consists of dollars, and a number of cents less than ten, there must be a cipher between the dollars and cents in place of dimes. Thus, 5 dollars and 4 cents must be written $5.04.

85. By examining the above table we see that 10 mills make 1 cent, and 100 cents, or 1000 mills one dollar; hence,

86. *To change dollars to cents,* multiply by 100; that is, annex two ciphers.

To change dollars to mills, annex three ciphers.

To change cents to mills, annex one cipher.

To change dollars and cents to cents, or dollars, cents and mills to mills, remove the decimal point and the sign $.

Exercises for the Slate.

1. Change \$196 to cents. Ans. 19600.
2. " \$1325 to mills. " 1325000.
3. " \$1.46 to cents. " 146.
4. " 56 cents to mills. " 560.
5. " \$19.425 to mills. " 19425.

87. *To change cents to dollars*, divide by 100; that is, point off two figures from the right.

To change mills to dollars, point off three figures.

To change mills to cents, point off one figure.

Exercises for the Slate.

1. Change 1967 cents to dollars. Ans. \$19.67.
2. " 1432 mills to " Ans. \$1.432.
3. In 34567 mills how many dollars? Ans. 34.567.
4. Reduce 3195 mills to dollars and cents. Ans. \$3.19½.

88. As the above currency is on the same principle as *decimal notation*, any operation, as addition, subtraction, multiplication, &c., may be performed upon it in the same manner as upon decimals.

89. Accounts are kept in sterling pounds, shillings and pence in Great Britain, Australia and New Zealand

90. *To reduce sterling pounds, shillings, pence, and farthings to Canada currency,*

TABLE.

		1 Farthing,	marked	$\frac{1}{4} = \frac{73}{144}$ C.
4 Farthings	make	1 Penny,	"	$d. = 2\frac{1}{36}$ "
12 Pence	"	1 Shilling,	"	$s. = 24\frac{1}{3}$ "
20 Shillings	"	1 Pound,	"	£ $= \$4.86\frac{2}{3}$

EXAMPLE.—Reduce £5 10s. 1¼d. to Canada currency

OPERATION.

```
£5 10s 1¼d
  = 5285 far.
        73
  --------
     15855
    36995
  --------
144)385805($26.79
```

ANALYSIS.—Since pounds, shillings and pence are composed of farthings, multiplying by 20, 12 and 4, reduces the whole amount to farthings = 5285 farthings. And since one farthing is equal to $\frac{73}{144}$ of a Canadian cent, 5285 farthings are equal to $5285 \times \frac{73}{144}$, (p. 38 ex. 1), or \$26.79. Hence,

RULE. Reduce pounds, shillings and pence sterling to farthings, and multiply by 73 and divide by 144. The quotient will be the equivalent in Canada currency.

NOTE 1.—In a final *remainder* reckon over ½ as a cent, less than ½ reject.

NOTE 2.—When there are only pounds in the exercise multiply by 486 2-3, the number of Canadian cents in a pound sterling. *See Appendix* II.

Mental Exercises.

1. How many Canadian cents are there in a three-penny piece ? in a four-penny piece ? in a sixpence ? in a shilling ?

2. How many Canadian dollars and cents are there in 2s, or a florin ? in 5 florins ? in 5s, or a crown ? in 10 crowns ? in 3 florins + 2 crowns ?

3. How many Canadian dollars and cents are there in 10s, or a half-sovereign ? in £1, or a sovereign ? in 10 sovereigns ? in £1 1s, or a guinea ? in 2 guineas + 3 half-sovereigns ?

Exercises for the Slate.

Reduce the following to Canadian currency :—

(1)	£1	3	6½	Ans. $5.73	(8)	£27	6	7¼	Ans $133.01
(2)	£11	11	6¾	$56.35	(9)	£26	16	8¾	$130.60
(3)	£44	15	7¾	$217.94	(10)	£10	11	4¾	$51.44
(4)	£26	18	9½	$131.11	(11)	£25	0	0	$121.67
(5)	£115	16	11¾	$563.80	(12)	£82	0	0	$399.07
(6)	£110	11	11½	$538.26	(13)	£64	0	0	$311.47
(7)	£365	4	5¼	$1777.41	(14)	£5	0	0	$24.33

91. *To reduce Canadian currency to pounds, &c., Stg.*

RULE. Reduce the dollars and cents to farthings by multiplying by 144 and dividing by 73. Reduce the farthings to pounds, shillings and pence. *See Appendix* II.

EXAMPLE.—Reduce $110.12½ to pounds, &c., stg

OPERATION.

$$\$110.12\tfrac{1}{2} \times 144 = 1585800 \div 73 = 4)\underline{21723} \text{ farthings}$$

$$12)\underline{5430} + \tfrac{3}{4}$$

$$2,0)\underline{45,2} + 6$$

$$22 + 12$$

Ans. £22 12s 6¾

NOTE.—For exercises under this rule the pupil may prove those of the former one.

REDUCTION OF LINEAR OR LONG MEASURE.

92. LONG MEASURE—TABLE.

12 Inches	make 1 Foot	marked	*ft.*
3 Feet	" 1 Yard	"	*yd.*
$5\frac{1}{2}$ Yards	" 1 Rod, Pole or Perch	"	*rd. or p*
40 Rods or Perches	" 1 Furlong	"	*fur.*
8 Furlongs	" 1 Mile	"	*m.*
3 Miles	" 1 League	"	*lea.*
$69\frac{1}{2}$ Miles (nearly)	" 1 Degree	"	*deg. or*

EXAMPLES.

1. In 18 po. 1 ft. 6 in. how many inches?

OPERATION.

18 po. 0 yd. 1 ft. 6 in.
$5\frac{1}{2}$
———
90
9
———
99 = yds. in 18 po.
3
———
298 = ft. in 18 po. 1 ft.
12
———
3582 = in. in 18 po. 1 ft. 6 in.

2. Reduce 5373 inches to poles, &c.

OPERATION.

12)5383
———
3)447 ft. 9 inches.
———
$5\frac{1}{2}$)149 yds.
2   2
———
11)298
———
27 po. $\frac{1}{2}$ yd. = 1 ft. 6 in
+ 9 in
———
27 po. 0 yd. 2 ft. 3 in

Mental Exercises.

1. How many inches are there in 3 ft.? in 5 ft.? in 10 ft.? in 12 ft. 4 in.?
2. How many feet are there in 4 yds.? in 6 yds.? in 9 yds.? in 15 yds.?
3. How many furlongs are there in 5 miles? in 6 m. 3 fur.? in 12 m. 7 fur?
4. In 100 inches how many yards, feet and inches?
5. At 9 dimes a foot, how many dollars will 4 yds. 2 ft. of iron railing cost?

Exercises for the Slate.

(1) Reduce 71280 in. to fur.
(2) " 3564 in. to po.
(3) " 63360 yds. to miles.
(4) " 570240 in. to miles.
(5) " 190080 ft. to miles.
(6) Reduce 36 po. 3 ft. to inches.
(7) " 45 m. 8 po. 1 yd. to yds.
(8) " 27 m. 1 po. $3\frac{1}{2}$ yd. to feet.
(9) " 72 m. 13 po. $\frac{1}{2}$ yd. to yds.
(10) " 74 m. 5 fur. 1 po. $\frac{1}{2}$ yd. to yds

11. In 9768042 inches how many miles?
Ans. 154 m. 1 fur. 13 po. 3 yds.

12. In 897682 yards how many miles?
Ans. 510 m. 0 fur. 14 po. 5 yds.

13. Reduce 103 m. 5 fur. 32 po. 5 yds. to feet.
Ans. 547683.

93. CLOTH MEASURE—TABLE.

$2\frac{1}{4}$ Inches	make	1 Nail.
4 Nails	"	1 Quarter, qr.
4 Quarters	"	1 Yard, 1 yd.
5 Quarters	"	1 English ell.
6 Quarters	"	1 French ell.
3 Quarters	"	1 Flemish ell.

EXAMPLES.

1. Reduce 27 yards 3 qr. to inches.

OPERATION.

27 yds. 3 qr.
4
———
111 = qrs. in 27 yds. 3 qr.
4
———
444 = nls. in 27 yds. 3 qr.
$2\frac{1}{4}$
———
888
111
———
999 = in. in 27 yds. 3 qr.

2. Reduce 153 nails to yds, &c.

OPERATION.

4)153
———
4)38 qrs. 1 nl.
———
9 yds. 2 qrs. 1 nl.

Mental Exercises.

1. How many inches are there in 3 nls.? in 2 qr. 1 nl.? in 2 yds. 1 nl.? in 5 qrs.?

2. How many quarters are there in 5 yds.? in 3 yds. 3 qrs.? in 6 yds. 2 qrs.?

3. How many yards are there in 5 qrs.? in 17 nls.? in 123 nls.? in 196 qrs.?

4. How many inches are there in 4 English ells? in 5 Flemish ells? in 19 French ells?

5 What is the cost of 3 French ells at 2 cents per inch?

Exercises for the Slate.

(1) Reduce 648 in. to yards.
(2) " 972 in. to Fl. ells.
(3) " 2268 in. to qrs.
(4) " 142 E. ells 4 qrs. to in.
(5) Reduce 3645 in. to E. ells.
(6) " 36 E. ells to inches.
(7) " 137 Fr. ells 3 qrs. to in.
(8) " 1215 in. to E. ells.

9. Reduce 127 yds. 3 qrs. 2 nls. to inches. Ans. 4603½.
10. In 39678 inches how many yards?
Ans. 1102 yds. 2 nls. 1½ in.
11. Reduce 426 English ells 3 qrs. to Flemish ells.
Ans. 711.

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94. REDUCTION OF SQUARE MEASURE.

TABLE.

144 Square inches	make	1 Square foot,	marked	*sq. ft.*
9 Square feet	"	1 Square yard,	"	*sq. yd.*
30¼ Square yards	"	1 Square pole,	"	*sq. po.*
40 Square poles	"	1 Square rood,	"	*ro.*
4 Roods	"	1 Acre,	"	*ac.*
640 Acres	"	1 Square mile,		

EXAMPLES.

1. Reduce 135 ac. 3 ro. 15 po. to poles.

OPERATION.

```
135 ac. 3 ro. 15 po.
  4
———
543 ro. in 135 ac. 3 ro.
 40
————
21735 po. in 135 ac. 3 ro. 15 po
```

2. Reduce 261414 yards to acres.

OPERATION.

```
 30¼)261414
  4      4
 ——  ———————
121
{11)1045656
    ———————
{11)  95059  7 } 95
      —————       — = 23¾
   4|0) 864|1 7 }  4
        —————
     4)216 ro. 1 po.
       ———          [yds.
    54 ac. 0 ro. 1 po. 23¾
```

Mental Exercises.

1. How many square feet are there in 6 square yards? in 19 yds. 3 feet? in 15 yds. 2 ft.?

2. How many acres are there in 880 poles? in 160 poles? in 320 poles? in 1240 poles?

3. At $4 per acre what will 920 poles of land cost?

4. Find the cost of 12 yards 3 feet at 7 dimes per foot.

Exercises for the Slate.

(1) Reduce 126 ac. 4 po. 5 yds. to yds.
(2) " 162 ac. 5 po. $10\frac{3}{4}$ yds. to yds.
(3) " 9 po. 9 in. to inches.
(4) " 90 ac. 18 yds. to yards.
(5) Reduce 1411380 in. to poles.
(6) " 304983 yds. to acres.
(7) 94 ac. 2 ro. 1 po. $5\frac{3}{4}$ yds. to yds.
(8) " 697104 yds. to acres.

9. In 36 ac. 3 ro. 28 po. 5 yds., how many feet?
Ans. 1608498.

10. Reduce 29 ac. 3 ro. 38 po. $15\frac{1}{2}$ yds. 8 feet to inches.
Ans. 188122032.

11. In $646376\frac{1}{2}$ feet how many acres?
Ans. 14 ac. 3 ro. 14 po. 6 yds. 1 foot.

REDUCTION OF CUBIC OR SOLID MEASURE.

95. SOLID MEASURE—TABLE.

1728	Cubic inches	make	1 Cubic foot, marked *cu. ft*
27	Cubic feet	"	1 Cubic yard, " *cu. yd.*
40	Cubic feet of rough or		
50	Cubic feet of hewn timber	"	1 Ton.
42	Cubic feet of timber	"	1 Ton.
128	Cubic feet	"	1 Cord of fire wood.
5	Cubic feet	"	1 Barrel bulk.

Exercises for the Slate.

1. In 125 cu. ft. 840 cu. in. how many cu. in.?
Ans. 216840.

2. Reduce 5224 cubic feet to cords. Ans. $40\frac{13}{16}$.

3. In a pile of wood 60 feet long, 20 feet wide, and 15 feet high, how many cords? Ans. $140\frac{5}{8}$.

4. A cellar is 32 feet long, 24 feet wide, and 6 feet deep, how much did it cost to dig it at 15 cents a cubic yard?
Ans. $25.60.

5. In a school-room 30 feet long, 20 feet wide and 10 feet high, with 50 pupils each breathing 10 cubic feet of air in one minute, in how long time will they breathe as mnch as the room contains? Ans. 12 min.

96. MEASURE OF CAPACITY—TABLE.

4 Gills (g)	make	1 Pint,	marked	*pt.*
2 Pints	"	1 Quart,	"	*qt.*
4 Quarts	"	1 Gallon,	"	*gal.*
2 Gallons	"	1 Peck,	"	*pk.*
4 Pecks	"	1 Bushel,	"	*bush.*
36 Bushels	"	1 Chaldron	"	*chal.*

EXAMPLES.

1. Reduce 27 bus. 1 pk. 1 gal. 1 qt. 1 pint to pints.

OPERATION.

27 bus. 1 pk. 1 gal. 1 qt. 1 pt.
4
———
109 pks.
2
———
219 gals.
4
———
877 qts.
2
———
1755 pints.

2. Reduce 594 gills to gallons.

OPERATION.

4)594
———
2)148 pts. 2 gills.
———
4)74 qts. 0 pts.
———
18 gals. 2 qts. 0 pts. 2 gills

NOTE.—As Liquid and Dry Measure are similarly divided, the above table and examples will answer both. (See Nova Scotia Table-book, pages 24 and 25.)

Mental Exercises.

1. How many gills are there in 4 pts.? in 3 qts. 3 pts.? in 6 qts. 3 pts. 1 gill?
2. How many quarts are there in 6 gals.? in 3 gals. 2 qts.? in 2 pks. 1 qt.?
3. How many gallons are there in 8 qts.? in 8 pts.? in 24 pts.? in 38 qts.?
4. What will be the cost of 7 gals. ' qts of burning fluid at 15 cents a quart?

Exercises for the Slate.

(1) Reduce 19 gals. 1 pt. to gills.
(2) " 11 pks. 1 gal. 1 qt. 3 gil. to gills.
(3) " 3 bus. 1 gal. 1 gill to gills.
(4) " 2 bus. 1 pk. 3 qt. 3 gils. to gills.
(5) Reduce 1942 bus. 1 qt. to qts.
(6) " 2880 gills to pks.
(7) " 18432 gills to bus.
(8) " 594 qts. to bush.

9. In 4983265 gills how many quarts ? Ans, 622908 qts. 1 gill.

10. Reduce 126 bus. 3 pks. 1 pt. to pints. Ans. 8113.

11. Reduce 1467896 quarts to chaldrons ? Ans. 1274 ch. 7 bus. 3 pks.

12. An innkeeper bought 50 bushels of oats at 65 cents a bushel, and retailed them at 25 cents a peck ; how much did he make on the lot ? Ans. $17.50.

REDUCTION OF WEIGHTS,

97. TROY WEIGHT—TABLE.

24 Grains	make	1 Pennyweight,	1 dwt.
20 Pennyweights	"	1 Ounce,	1 oz.
12 Ounces	"	1 Pound,	1 lb.

This weight is used in weighing the precious metals and stones ; also in scientific investigations.

EXAMPLES.

1. Reduce 31 lbs, 10 oz. 8 dwts. 12 grs. to grains.

OPERATION.

```
    31 lbs. 10oz. 8dwt. 12grs.
    12
   ---
   382 oz.
    20
  ----
  7648 dwt.
    24
 -----
 30604
15296
------
183564 grains.
```

2. Reduce 28197 dwt. to lbs.

OPERATION.

```
2|0)2819|7
   -------
 12)1409 oz. 17 dwt.
    -------
     117 lbs. 5 oz. 17 dwt.
```

Mental Exercises.

1. How many grains are there in 5 dwts. ? in 6 dwts. 7 grains ? in 15 dwts. 3 grs. ?

2. How many ounces are there in 120 dwt. ? in 200 dwt. ? in 240 dwts. ?

3. What will a gold chain weighing 9 dwt. 15 grs. cost at 3 cents a grain ?

4, What is the value of a silver cup, weighing 5 oz. 4 dwts. at 15 cents per pennyweight?

5. In 5 ingots of gold, each weighing 9 oz. 5 dwt. how many dwts.?

Exercises for the Slate.

(1) Reduce 9 oz. 12 dwt. 18 grs. to grs.	(5) Reduce 207396 grs. to lbs.
(2) " 1 lb. 1 oz. 19 dwts. to grs.	(6) " 4338 dwts. to lbs.
(3) " 1 lb. 3 oz. 9 dwt. to grs.	(7) " 155520 grs. to lbs.
(4) " 20 lbs, 10oz. 18dwts.to dwts.	(8) " 17280 dwts. to lbs.

9. Reduce 37 lbs. 11 oz. 19 dwts. to dwts.
Ans. 9119 dwts.

10. Reduce 87 lbs. 19 grs· to grains. Ans. 501139.

11. Rednce 578096 grains to pounds.
Ans. 100 lbs. 4 oz. 7 dwts. 8 grs.

12. A miner had 14 lbs. 10 oz. 18 dwt. of gold dust: how much was it worth at 75 cents a dwt.? Ans. $2683.50.

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98. APOTHECARIES' WEIGHT—TABLE.

20 Grains	make	1 Scruple,	1 *sc.*	or ℈
3 Scruples	"	1 Dram,	1 *dr.*	or ʒ
8 Drams	"	1 Ounce,	1 *oz.*	or ℥
12 Ounces	"	1 Pound,	1 *lb.*	or ℔

NOTE. Apothecaries and Physicians mix their medicine by this weight, but they buy and sell by Avoirdupois.

Exercises for the Slate.

(1) Reduce 9 lbs. 1 oz. 1 dr. to grs.	(5) Reduce 63 lbs. 1 dr. 3 grs. to grs.
(2) " 18 lbs. 6 drs. to scr.	(6) " 84 lbs. 7 oz. 7 drs. to grs.
(3) " 36 lbs. 1 scr. 16 grs. to grs.	(7) " 207360 grains to lbs.
(4) " 45 lbs. 2 scr. 5 grs. to grs.	(8) " 259200 grains to lbs.

10. Reduce 47℔. 6℥. 4ʒ. to scruples. Ans. 13692 scr.

11. How many pounds of medicine would a physician use in 365 days, if he averaged daily 5 prescriptions of 20 grains each? Ans. 6℔. 4℥. 1℈.

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99. AVOIRDUPOIS WEIGHT—TABLE.

16 Drams	make	1 Ounce,	marked	1 *oz.*
16 Ounces	"	1 Pound,	"	1 *lb.*
28 Pounds	"	1 Quarter,	"	1 *qr.*
4 Quarters	"	1 Hundredweight		1 *cwt.*
20 Hundredweight	"	1 Ton,	"	1 *ton.*

NEW SYSTEM OF WEIGHT.

The different units are the same as in the old system, thus

16 Drams	make	1 Ounce,	marked 1 *oz.*
16 Ounces	"	1 Pound,	" 1 *lb.*
25 Pounds	"	1 Quarter,	" 1 *qr.*
4 Quarters	"	1 Hundredweight	1 *cwt.*
20 Hundredweight	"	1 Ton,	" 1 *ton.*

NOTE.—The old system of weight is called long, and the new system short weight.

EXAMPLES.

1. Reduce 81 cwt. 2qrs. 25 lbs., long weight, to pounds.

OPERATION.

```
81 cwt. 2 qrs. 25 lbs.
 4
———
326 qrs.
 28
———
2633
652
———
9153 lbs.
```

Or,

```
81 cwt. 2 qrs. 25 lbs.
8100 = 81 × 100
 972 = 81 × 12
  56 = pounds in 2 qrs.
  25 =    "      given.
9153 =    "      required.
```

2. Reduce 72 cwt. 2 qrs. 22 lbs., short weight, to pounds.

OPERATION.

```
72 cwt. 2 qr. 22 lbs.
 4
———
290 qrs.
 25
———
1472
580
———
7272 lbs.
```

Or,

```
72 cwt. 2 qrs. 22 lbs
7200 = pounds in 72 cwt.
  50 =    "    "  2 qrs.
  22 =    "    given.
7272 =    "    required.
```

Mental Exercises.

1. How many ounces are there in 3 lbs.? in 5 lbs. 10 oz.? 6 lbs. 13 oz.?
2. In 3 cwt. 5 lbs. short weight, how many pounds? How many ounces?
3. What will 1 ton 5 cwt. of hay cost, if 5 cwt. cost $3?
4. What will 2 cwt. 12 lbs., short weight, of beef cost at 6 cents a pound?
5. If 8 ounces of tea cost 40 cents, what is the cost of 2 lbs.?

Exercises for the Slate.

1. Reduce 8 cwt. 2 qrs. 19 lbs. 4 oz. 12 drs., long weight, to drs.
2. " 1 ton 2 cwt. 3 qrs. 7 lbs. 9 oz. 13 drs., long weight, to drs.
3. " 22 tons 13 cwt. 1 qr. 5 lbs. 9 oz., long weight, to drs.
4. " 25 tons 2 cwt. 1 qr. 13 oz., long weight, to oz.
5. " 42 tons 14 cwt. 2 qrs. 3 lbs. 5 oz., short weight, to ounces.
6. " 7 cwt. 1 qr. 4 lbs. 7 oz. 5 drs., short weight, to drs.
7. " 6939 drams to pounds.
8. " 1032228 drams to cwt., long weight.
9. " 3 qrs. 15 lbs. 15 oz. 15 drs., long weight, to drs. Ans. 25599 drs.
10. " 94 tons 19 cwt. 2 qrs. 24 lbs. 10 oz. 15 drs., long weight, to drams. Ans. 54468783.
11. " 493865 lbs. to tons, long weight. Ans. 220 tons 9 c. 2 qr. 1 lb.
12. " 204250 oz. to cwt., short weight. Ans. 127 cwt. 2 qr. 15 lb. 10 oz.

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100. REDUCTION OF TIME.

TABLE.

1 Second is written thus: 1″

60 Seconds	make 1 Minute,	marked 1′.
60 Minutes	" 1 Hour,	" 1 *hr.*
24 Hours	" 1 Day,	" 1 *day.*
7 Days	" 1 Week,	" 1 *wk.*
28 Days	" 1 Lunar month.	
28, 29, 30, or 31 Days	" 1 Calendar month.	
12 Calendar months	" 1 Year.	
365 Days	" 1 Common year.	
366 Days	" 1 Leap year.	

Mental Exercises.

1. How many seconds are there in 3 hrs.? in 4 hrs. 20′? in 5 hrs. 9″?
2. How many hours are there in 4 days 5 hrs.? in 2 wks, 3 days 12 hrs.?
3. How many weeks are there in 72 days? in 85 days? in 63 days?

4. How many days are there from April 15th to August 10th inclusive?

Exercises for the Slate.

REDUCE

(1) 18 days 27 min. 18 sec. to sec.
(2) 27 days 36 min. 27 sec. to sec.
(3) 720 d. 11 h. 37 min. 30 sec. to sec.
(4) 36 yrs. 9 hrs. 36 min. to min.
(5) 9 yrs. 2 hrs. 45 min. 9 sec. to sec.
(6) 365 dys. 5 hrs. 48 min. 45 sec. to sec.
(7) 8 yrs. 5 days 45 min. to seconds.
(8) 283824000 sec. to years.
(9) 9460800 min. to years.
(10) 103680 min. to days.

11. Reduce 48 days 17 sec. to seconds. Ans. 4147217 sec.

12. Reduce 53 days 23 hrs. 26 min. to minutes. Ans. 77726 min.

13. How many times does a clock pendulum, beating seconds, vibrate in one day? Ans. 86400.

14. How much time will a person gain in 30 years, by rising, each day, 42 minutes earlier than his usual time? Ans. 319 days 9 hours.

MISCELLANEOUS TABLE.

12	individual things	make	1 dozen.
12	dozen	"	1 gross.
12	gross	"	1 great gross.
20	individual things	"	1 score.
24	sheets of paper	"	1 quire.
20	quires	"	1 ream.
112	pounds	"	1 quintal.
200	"	"	1 barrel of pork or beef.
196	"	"	1 barrel of flour.
14	"	"	1 stone.

Exercises for the Slate.

1. In 365 gross 11 doz. 9 units, how many individual things? Ans. 52701.

2. A person bought 219 cwt. 2 qrs. 2 lbs., short weight, of codfish at \$5 a quintal, what did the whole amount to? Ans. \$980.00.

3. What will 6 tons 6 cwt., long weight, of flour cost at \$7.75 a barrel? Ans. \$558.00.

4. What will 15 reams of paper cost at one cent per sheet? Ans. \$72.00.

5. It is said Mr. Jos. Gillott, of Birmingham, manufactures annually 150 millions of different kinds of pens; how many boxes will it require to hold them, each box holding one gross? Ans. 1041666 and 8 doz. pens over.

COMPOUND ADDITION.

101. Compound Addition is the method of collecting several numbers of the same kind, but containing different denominations of that kind into one number.

102. *To Add Compound Numbers.*

EXAMPLE.—A merchant paid £16 3s. $9\frac{1}{2}$d. for tea; £46 11s. $1\frac{1}{4}$d. for sugar; £101 3s. 5d. for flour; £13 14s. $2\frac{1}{4}$d. for molasses, and £108 11s. $4\frac{3}{4}$d. for dry goods; what was the amount of his bill?

OPERATION.

£	s.	d.
16	3	$9\frac{1}{2}$
46	11	$1\frac{1}{4}$
101	3	5
13	14	$2\frac{1}{4}$
108	11	$4\frac{3}{4}$
£286	3	$10\frac{3}{4}$

ANALYSIS.—Arranging the numbers in columns, placing units of the *same* denomination under each other, we first begin at the right hand column, or lowest denomination, and find the amount to be 7 far., which is equal to 1 penny 3 farthings. We write the farthings under the column of farthings, and add the 1 penny to the column of pence. We find the amount of the second column, (with the 1 penny added), to be 22 pence, which is equal to 1 shilling and 10 pence. Writing the 10 pence under the column of pence, we add the 1 shilling to the next column. Adding this column as the preceding ones, we find the amount to be 43 shillings, which is equal to £2 and 3 shillings. Placing the 3s. under the column of shillings, we add £2 to the column of pounds. Adding this last column, we find the amount to be £286, and the whole result, or answer to be £286 3s. $10\frac{3}{4}$. Hence,

RULE. I. Write the numbers so that those of the same unit value will stand in the same column.

II. Beginning at the right hand, add each denomination as in simple numbers, carrying to each succeeding denomination one for as many units as it takes of the denomination added, to make one of the next higher denomination.

Mental Exercises.

1. Add together $5\frac{1}{4}$d., $6\frac{1}{4}$d., $3\frac{1}{2}$d., and 2s. 6d$\frac{1}{4}$d.
2. Find the sum of 1s. 2d., 1s. $3\frac{1}{2}$d., 4s. $6\frac{1}{4}$d.
3. A farmer sold 4 bundles of hay, weighing as follows, 1st, 2 cwt. 3 qrs., 2nd, 1 cwt. 2 qrs. 14 lbs., 3rd, 1 cwt. 3 qr., and the 4th, 2 cwt. 0 qr. 14 lbs.; what was the weight of the whole?

Exercises for the Slate.

(1)

£	s.	d.
2	16	9
8	17	6
8	18	5
9	5	11

(2)

£	s.	d.
2	7	8
3	14	5
9	10	7
9	2	10

(3)

£	s.	d.
2	10	7¾
7	16	10
9	14	9¼
8	15	8

(4)

£	s.	d.
29	9	10¼
25	18	4½
76	16	11¾
94	14	3

(5)

£	s.	d.
3	10	5½
7	13	4¾
6	12	8½
4	9	6¼
5	13	5¼
5	18	4¾
4	16	6

(6)

lbs.	oz.	dr.
33	10	7
37	8	13
78	12	8
65	14	5
26	6	10
81	13	8
14	7	11

(7)

cwt.	qr.	lb.
31	2	23
27	1	16
49	0	8
57	3	12
79	2	6
50	3	20
32	0	16

(8)

tons.	cwt.	qr.
3	17	2
1	13	0
5	8	3
6	12	1
7	13	2
4	11	3
2	17	2

(9)

oz.	dwt.	grs.
35	12	21
64	17	19
48	16	11
65	18	4
51	13	23
98	19	14

(10)

oz.	dr.	scr.
35	5	2
38	2	1
75	6	0
47	7	2
89	4	1
52	1	2

(11)

yds.	ft.	in.
35	2	10
34	0	6
69	2	8
42	1	11
35	2	7
60	1	8
56	1	5

(12)

yd.	qrs.	nl.
38	2	3
45	1	2
37	0	3
72	3	1
42	2	2
67	3	1
42	0	3

(13)

m.	fur.	po.
36	6	33
67	4	16
63	5	9
28	6	25
84	2	8
35	4	31
51	7	15

(14)

fur.	po.	yds.
35	26	3½
74	35	2½
57	17	5
46	8	4½
65	14	3
12	22	0½
83	31	1

(15)

ac.	ro.	po.
37	1	35
25	2	18
68	1	36
34	3	15
46	1	13
50	1	0
63	3	22

(16)

ac.	ro.	po.
24	3	7
76	1	38
15	2	23
53	3	19
40	0	34
17	1	1
49	1	37

17. Find the sum of 34 lb. 6 oz. 14 dwt., 53 lbs. 10 oz. 5 dwt., 76 lb. 4 oz. 12 dwt., 38 lb. 8 oz. 10 dwt., 83 lb. 11 oz 18 dwt., 67 lb. 5 oz. 7 dwt.

18. Find the sum of 37 dr. 1 scr. 16 grs., 24 dr. 12 grs., 69 dr. 2 scr. 7 grs., 45 dr. 1 scr. 13 grs., 58 dr. 2 scr. 19 grs., 89 dr. 1 scr. 6 grs.

19. Find the sum of 31 da. 17 h. 53 m., 25 da. 21 h. 39 m., 72 da. 8 h. 16 m., 66 da. 23 h. 45 m., 74 da. 7 h. 23 m., 55 da. 15 h. 44m.

20. A farmer has 23 ac. 1 ro. 26 po. in wheat, 45 ac. 2 ro. 31 po. in oats, 24 ac. 1 ro. 17 po. in barley, 87 ac. 3 ro. 15 po. in grass, and 65 ac. 2 ro. 23 po. in wood land, how much has he altogether?

21. Find the sum of 79 m. 7 fur. 24 po. 4 yd. 2 ft. 7 in., 58 m. 3 fur. 34 po. 3 yd. 1 ft. 10 in., 61 m. 6 fur. 23 po. 2 yd. 2 ft. 8 in., 97 m. 5 fur. 39 po. 5 yd. 1 ft. 9 in., 25 m. 3 fur. 24 po. 1 yd. 0 ft. 11 in. Ans. 323 m. 3 fur. 27 po. 1 yd. 2 ft. 3 in

22. Add together 324 tons 19 cwt. 2 qrs., 264 tons 14 cwt. 15 lbs., 98 tons 3 qrs. 16 lbs. 14 oz., 981 tons 13 oz. 15 drs., long weight. Ans. 1668 tons 14 cwt. 2 qrs. 4 lbs. 11 oz. 15 drs.

23. A farmer received 60 cents a bushel for 4 loads of oats weighing as follows: 2385, 2761, 3962, and 1500 pounds; how many bushels were there, and what was the whole amount? Ans. 312 bus. $187.20.

24. Find the sum of 23 bus. 3 pks. 7 qts. 1 pt., 34 bus. 2 pk. 1 pt., 42 bus. 3 pk. 5 qt., 51 bus. 1 pk. 4 qt. 1 pt., 23 bus. 3 qt., 11 bus. 3 pk. 4 qt. Ans. 187 bus. 3 pks. 1 pt.

25. A man in digging a cellar removed 163 cu. yds. 26 cu. ft. of earth; in digging a trench 19 cu. yds. 14 cu. ft.; and in digging a cistern 17 cu yds. 14 cu. ft.; what was the amount of earth removed, and what did it cost at 22 cents per cubic yard? Ans. 201 cu. yd. $44.22.

COMPOUND SUBTRACTION.

103. Compound Subtraction is the method of finding the difference between two numbers of the same kind containing different denominations of that kind.

104. *To subtract compound numbers.*

EXAMPLE.—A merchant bought 15 cwt. 3 qrs. 14 lb. (long weight) of sugar and sold 9 cwt. 2 qrs. 18 lbs.; how much had he left.

OPERATION.		
cwt.	qrs.	lbs.
15	3	14
9	2	18
Ans. 6	0	24

ANALYSIS.—Writing the subtrahend under the minuend, placing units of the same denomination under each other, we begin at the right hand, or lowest denomination; since we cannot take 18 lbs. from 14 lbs., we add 1 qr. or 28 lbs., to 14 making 42 lbs.; and taking 18 lbs. from 42 lbs., we write the remainder, 24 lbs., underneath the column of pounds. Having added 1 qr. or 28 lbs. to the minuend, we now add 1 qr. to the 2 qrs. in the subtrahend, making 3 qrs.; and 3 qrs. from 3 qrs. leaves 0 qrs., which we write in the remainder, under the column of quarters. Lastly, we take 9 cwt. from 15 cwt. and write the remainder, 6 cwt., under the column of hundreds weight. Hence,

RULE. I. Write the subtrahend under the minuend, so that units of the same denomination shall stand under each other.

II. Beginning at the right hand, subtract each denomination separately, as in simple numbers.

III. If the number of any denomination in the subtrahend exceed that of the same denomination in the minuend, add to the number in the minuend as many units as make one of the next higher denomination, and then subtract; in this case add 1 to the next higher denomination of the subtrahend before subtracting. Proceed in the same manner with each denomination.

Mental Exercises.

From $3\frac{1}{2}$d. take $1\frac{3}{4}$d.; 1s. 9d. take 11d.; 2s. $9\frac{1}{2}$d. take 1s. $6\frac{1}{2}$d.

2. A man having 4 ac. 2 ro. of land sold 1 ac. 3 ro., how much land had he left?

3. A person having £3 6s. 3d., bought 14s. 8d. worth of tea, how much money was left after paying for it?

4. A miner having 5 dwt. 12 grs. of gold, sold 2 dwt. 20 grs., how much had he left?

Exercises for the Slate.

SECTION I.

	£	s.	d.	£	s.	d.		£	s.	d.	£	s.	d.
(1)	40	15	3—	13	9	11	(9)	147	0	$0\frac{3}{4}$—	29	16	$8\frac{1}{2}$
(2)	77	12	5—	13	19	11	(10)	365	1	11 —	139	16	$10\frac{1}{2}$
(3)	95	10	0—	13	13	10	(11)	558	13	$1\frac{1}{4}$—	216	4	$8\frac{1}{2}$
(4)	120	9	5—	47	15	1	(12)	721	2	6 —	387	15	$11\frac{1}{4}$
(5)	94	10	6—	39	19	10	(13)	185	2	1 —	67	18	$8\frac{3}{4}$
(6)	92	0	7—	46	11	7	(14)	526	1	$1\frac{1}{4}$—	318	19	$8\frac{3}{4}$
(7)	82	14	1—	0	17	11	(15)	381	5	$7\frac{3}{4}$—	11	11	11
(8)	100	0	0—	0	0	4	(16)	980	7	$2\frac{1}{4}$—	583	7	$11\frac{1}{2}$

SECTION II.

The following exercises are to be worked as the given example.

NOTE.--1. The teacher may require the pupil after finishing the subtraction in each exercise, to add all the lines together.

	£	s.	d.	
EXAMPLE.	10	18	$2\frac{3}{4}$	—Minuend.
	6	10	$11\frac{1}{4}$	—Subtrahend.
	4	7	$3\frac{1}{2}$	=2nd line subtracted from first.
	2	3	$7\frac{3}{4}$	=3rd " " " second.
	2	3	$7\frac{3}{4}$	=4th " " " third.
	£26	3	9	sum=12 times 5th line.

	s.	d.		s.	d.
(1)	1	$10\frac{1}{2}$	—	1	$1\frac{1}{2}$
(2)	2	$2\frac{1}{4}$	—	1	$3\frac{3}{4}$
(3)	3	$2\frac{3}{4}$	—	1	$11\frac{1}{4}$
(4)	11	$10\frac{1}{2}$	—	7	$1\frac{1}{2}$

	£	s.	d.		£	s.	d.
(5)	3	15	$6\frac{1}{4}$	—	2	5	$3\frac{3}{4}$
(6)	4	19	$10\frac{3}{4}$	—	2	19	$11\frac{1}{4}$
(7)	5	17	$7\frac{1}{4}$	—	3	10	$6\frac{3}{4}$
(8)	6	18	$5\frac{1}{4}$	—	4	3	$0\frac{3}{4}$

	yds.	ft.	in.		yds.	ft.	in.
(9)	19	1	9	—	11	2	3
(10)	23	0	7	—	13	2	9
(11)	44	2	$9\frac{1}{2}$	—	26	2	$10\frac{1}{2}$
(12)	70	9	$0\frac{3}{4}$	—	42	5	$5\frac{1}{4}$

	yds.	qrs.	nls.		yds.	qrs.	nls.
(13)	79	2	3	—	47	3	1
(14)	112	3	1	—	67	2	3
(15)	634	1	$3\frac{1}{4}$	—	380	2	$2\frac{3}{4}$
(16)	69	3	$2\frac{3}{4}$	—	41	3	$3\frac{1}{4}$

	m.	fur.	po.	yds.		m.	fur.	po.	yds
(17)	57	2	28	$3\frac{1}{2}$	—	34	3	9	1
(18)	61	6	18	1	—	37	0	26	5
(19)	44	6	33	$4\frac{3}{4}$	—	26	7	12	$1\frac{3}{4}$
(20)	16	4	4	$0\frac{1}{2}$	—	9	7	10	$2\frac{1}{2}$

	ac.	ro.	po.		ac.	ro.	po.
(21)	74	1	20	—	44	2	20
(22)	44	3	35	—	26	3	37
(23)	284	1	15	—	170	2	17
(24)	131	3	$12\frac{1}{2}$	—	79	0	$15\frac{1}{2}$

	bus.	pks.	gals.		bus.	pks.	gals
(25)	74	1	1	—	44	2	1
(26)	83	0	1	—	49	3	1
(27)	602	3	$0\frac{1}{2}$	—	361	2	$1\frac{1}{2}$
(28)	301	3	1	—	181	0	1

	℔	℥	ʒ	℈	grs.		℔	℥	ʒ	℈	grs.
(29)	114	11	7	2	10	—	68	11	7	2	14
(30)	73	8	0	2	0	—	44	2	3	1	16
(31)	90	2	5	0	15	—	54	1	4	2	5

32. From 546 lbs. 10 oz. 2 dwt. 8 grs. take 397 lbs. 11 oz. 15 dwt. 14 grs. Ans. 148 lbs. 10 oz. 6 dwt. 18 grs.

33. From 486 years take 395 years 8 mo. 3 wks. 5 days. Ans. 90 yrs. 3 mo. 2 days.

34. From 310 tons 13 cwt. 2 qrs., long weight, take 77 tons 13 cwt. 1 qr. 14 lbs. four times. Ans. 0.

35. From 481 acres 1 ro. 18 po. 11 yds. take 120 ac. 1 ro. 14 po. 18 yds. four times. Ans. 0.

36. What is the difference between 198 m. 7 fur. 25 po. 2 yd. 1 ft. 10 in. and 300 miles?
Ans. 101 m. 14 po. 2 yd. 2ft. 8 in.

37. A person having 63 gallons of wine, drank, on an average, for five years, including two leap years, one gill of wine a day; how much remained?
Ans. 5 gals. 3 qts. 1 pt. 1 gill.

38. A man having dug from a trench 126 cub. yds. 16 cub. ft., from a cistern 18 cu. yd. 18 cu. ft. 196 cu. in., and from other places 126 cu. yd. 26 cu. ft., was paid for 196 cu. yd. 26 cu. ft. 1714 cu. in.; how much remained unpaid?
Ans. 75 cub. yd. 6 cub. ft. 210 cub. in.

COMPOUND MULTIPLICATION.

105. Compound Multiplication is the method of multiplying a quantity consisting of several denominations by a given number.

106. *To Multiply a Compound Number.*

CASE I.

107. *When the multiplier is under* 12.

EXAMPLE 1.—A man sold 6 lots of land, each lot containing 4 ac. 2 ro. 14 po.: how much land is there in all?

OPERATION.

ac.	ro.	po.
4	2	14
		6
27	2	4

ANALYSIS.—In 6 lots there are 6 times as much land as in 1 lot. We write the multiplier under the lowest denomination of the multiplicand, and proceed thus; 6 times 14 po. are 84 poles, equal to 2 ro. 4 po.; and we write the 4 po. under the number multiplied, and carry the 2 ro. to the next product. Then, 6 times 2 ro. are 12 ro., and 2 ro. added make 14 ro., equal to 3 ac. 2 ro.; and we write the 2 ro. under the number multiplied. Again, 6 times 4 ac. are 24 ac., and 3 ac. added make 27 ac., which we write under the number multiplied.

From the above example and illustration we deduce the following general rule:

RULE. I. Write the multiplier under the lowest denomination of the multiplicand.

II. Multiply as in simple numbers, and carry as in addition of compound numbers.

Mental Exercises.

1. Find the cost of 5 lbs. of tea at 3s. 9d. per pound.
2. What will 9 lbs. of coffee cost at 1s. 6d. per pound ?
3. What will 36 pairs of stockings cost at 3s. $1\frac{1}{2}$d. per pair ?
4. How many acres are there in four fields each containing 2 ac. 3 ro. 10 po. ?
5. If a tailor requires 3 yds. 1 qr. 1 nl. of cloth to make a coat, how many yards must he have to make five coats of the same size ?

Exercises for the Slate.

SECTION I.

EXAMPLE.—Multiply £1 2s. $9\frac{1}{4}$d. by 4, and £8 7s. $2\frac{3}{4}$d. by 4.

OPERATION.

£	s.	d.
1	2	$9\frac{1}{4}$
		4
£4	11	1

OPERATION.

£	s.	d.
8	17	$2\frac{3}{4}$
		4
£35	8	11

	£	s.	d.
Test	4	11	1
	35	8	11
	40	0	0

Multiply each of the following couplets by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. Multiplying them *all* first by 2, then *all* by 3, then *all* by 4, &c., testing them as above.

	s.	d.		s.	d.
(1)	2	3	and	17	9
(2)	3	4	and	16	8
(3)	4	$5\frac{3}{4}$	and	15	$6\frac{1}{4}$
(4)	7	$9\frac{1}{4}$	and	12	$2\frac{3}{4}$
(5)	6	$8\frac{1}{2}$	and	13	$3\frac{1}{2}$

	£	s.	d.		£	s.	d.
(6)	4	3	$9\frac{1}{2}$	and	5	16	$2\frac{1}{2}$
(7)	3	12	$8\frac{3}{4}$	and	6	7	$3\frac{1}{4}$
(8)	8	19	$11\frac{3}{4}$	and	1	0	$0\frac{1}{4}$
(9)	5	17	$6\frac{1}{2}$	and	4	2	$5\frac{1}{2}$
(10)	6	13	$9\frac{3}{4}$	and	3	6	$2\frac{1}{4}$

	ac.	ro.	po.	yds.		ac.	ro.	po.	yds.
(11)	2	3	21	16	and	7	0	18	$14\frac{1}{4}$
(12)	5	3	24	19	and	4	0	15	$11\frac{1}{4}$
(13)	3	2	17	3	and	6	1	22	$27\frac{1}{4}$
(14)	6	0	27	15	and	3	3	12	$15\frac{1}{4}$

	yds.	qrs.	nls.		yds.	qrs.	nls.
(15)	3	3	3	and	6	0	1
(16)	7	2	1	and	2	1	3
(17)	8	1	1	and	1	2	3
(18)	9	2	$1\frac{1}{2}$	and	0	1	$2\frac{1}{2}$

CASE II.

108. *When the Multiplier is a Composite number.*

EXAMPLE.—What is the weight of 42 bundles of hay each weighing 3 cwt. 2 qrs. 12 lbs, (short weight) ?

OPERATION.

cwt.	qr.	lbs.	
3	2	12	
		6	
21	2	22	weight of 6 bundles.
		7	
152	0	4	weight of 42 bundles.

ANALYSIS.—Multiplying the weight of 1 bundle by 6, we obtain the weight of 6 bundles, and the weight of 6 bundles multiplied by 7, gives the weight of 42 bundles.

SECTION II.

EXAMPLE.—Multiply £46 13s. $10\frac{1}{2}$d., and £53 6s. $1\frac{1}{2}$d., by 48.

OPERATION,

£	s.	d.
46	13	$10\frac{1}{2}$
		6
280	3	3
		8
£2241	6	0

OPERATION.

£	s.	d.
53	6	$1\frac{1}{2}$
		12
639	13	6
		4
£2558	14	0

	£	s.	d.
Test,	2241	6	0
	2558	14	0
	£4800	0	0

Multiply each of the following couplets by 14, 16, 18, 20, 21, 22, 24, 27, 28, 30, 32, 36, 40, 42, 45, 48, 50, 54, 56, 60, 64, 72, 81, 96, testing the products as above.

	£	s.	d.		£	s.	d.
(1)	89	13	$6\frac{3}{4}$	and	10	6	$5\frac{1}{4}$
(2)	72	14	$3\frac{1}{2}$	and	27	5	$8\frac{1}{2}$
(3)	36	10	$11\frac{1}{4}$	and	63	9	$0\frac{3}{4}$

	lbs.	oz.	dr.		lbs.	oz.	dr.
(4)	19	14	14	and	80	1	2
(4)	89	15	11	and	10	0	5
(6)	72	13	$3\frac{1}{4}$	and	27	2	$12\frac{3}{4}$

(LONG WEIGHT.)

	tons	cwt.	qrs.	lbs.		tons	cwt.	qrs.	lbs
(7)	83	15	3	27	and	16	4	0	1
(8)	72	16	2	$22\frac{1}{2}$	"	27	3	1	$5\frac{1}{2}$
(9)	91	18	3	$11\frac{1}{4}$	"	8	1	0	$16\frac{3}{4}$
(10)	54	15	2	$27\frac{3}{4}$	"	45	4	1	$0\frac{1}{4}$

(SHORT WEIGHT.)

	cwt.	qrs.	lbs.		cwt.	qrs.	lbs.
(11)	72	3	22	and	27	0	3
(12)	91	1	24	"	8	2	1
(13)	12	3	$19\frac{1}{2}$	"	87	0	$5\frac{1}{2}$
(14)	87	1	$22\frac{1}{2}$	"	12	2	$2\frac{1}{2}$

Multiply each of the above by 100, 110, 120, 121, 132, 144, using two factors, and by 112, 144, 420, 441, 504, using three factors.

CASE III.

109. *When the multiplier cannot be reduced to factors.*

EXAMPLE.—How many bushels of oats in 47 barrels, each containing 3 bus. 1 pk.?

OPERATION.

$47 = (5 \times 9) + 2$

bus.	pks.	
3	1 × 2	
	5	
16	1	in 5 barrels.
	9	
146	1	in 45 barrels.
6	2 "	2 "
152	3	in 47 barrels.

ANALYSIS.—Multiplying the contents of 1 barrel by 5, and the resulting product by 9, we have the contents of 45 barrels, which is the composite number *next less* than the given prime number 47. Next multiplying the contents of 1 barrel by 2, we have the contents of 2 barrels, which added to the contents of 45 barrels, gives us the contents of $45 + 2 = 47$ barrels.

SECTION III.

Multiply each of the following couplets by 19, 29, 31, 43. 67, 76, 83, 91, 97, 111, 113. 127, 131, 143, 139, 174, 345, 461, 783, 199, 911, 888 and test the results as in the preceding section.

	bus.	pks.	gal.	qts.	pts.		bus.	pks.	gal.	qts.	pts.
(1)	135	3	1	3	1	and	864	0	0	0	1
(2)	635	1	0	2	1	and	364	2	1	1	1
(3)	299	0	1	1	1	and	700	3	0	2	1

SECTION IV.

110. Take any couplet, as in any of the preceding sections. Take *any* multiplier. Prefix to the couplet any two numbers whose sum is *one* less than the multiplier chosen. Multiply both the multiplicands thus formed by the multiplier chosen, and add the products.

EXAMPLE 1.—Take the couplet £16 13s. 9d. and £83 6s. 3d. Take 8 as multiplier. Prefix to the couplet 7, (8 — 1). Then multiply by 8. (7 = 4 + 3)

OPERATION.

£416	13	9
		8
£3333	10	0

OPERATION.

£383	6	3
		8
£3066	10	0

£3333	10	0
£3066	10	0
£6400	0	0 = $8^2 \times 100$

EXAMPLE 2.—Take the couplet 196 cwt. 2 qrs. 27 lbs. and 803 cwt. 1 qr. 1 lbs., long weight, and 48 as multiplier. Prefix 47, = (29 + 18), and multiply as before.

OPERATION.

cwt.	qrs.	lbs.
29,196	2	27
		6
175180	1	22
		8
1401443	2	8

OPERATION.

cwt.	qrs.	lbs.
18,803	1	1
		6
112819	2	6
		8
902556	1	20

cwt.	qrs.	lbs.	
1401443	2	8	
902556	1	20	
2304000	0	0	$= 48^2 \times 1000$

SECTION V.

Find the value of—

1. 37 tons 13 cwt. 3 qrs. 12 lbs., long weight, $\times$ 6
Ans. 226 tons 3 cwt. 16 lbs.
2. 39 m. 7 fur. 28 po. 4 yds. $\times$ 6.
Ans. 239 m. 6 fur. 12 po. 2 yd.
3. 92 yd. 3 qr. 1 nl. 2 in. $\times$ 765. Ans. 71044 yd. 0 qr. 1 nl.
4. 27 y. 54 days 15 h. 29 m. $\times$ 921.
Ans. 25004 y. 323 d. 4 h. 9 m.
5. If 1 acre of land produce 45 bus. 3 pks. 6 qts. 1 pt. of corn, how much will 64 acres produce? Ans. 2941 bus.
6. If \$80 purchase 4 ac. 3 ro. 26 po. 20 sq. yd. 3 sq. ft. of land, how much will \$4800 buy? Ans. 295 ac. 10 sq. yd.
7. What will 16 tons of hay cost at £3 19s. 6½d. per ton?
Ans. £63 12s. 8d.
8. What is the cost of 8 bus. 3 pks. of beans at 5½ per quart? Ans. £6 8s. 4d.
9. If 1 pt. 3 gills of wine fill 1 bottle, how much will be required to fill a great gross of bottles of the same capacity?
Ans. 378 gals.

10. Windsor, March 17th, 1866.

MR. C. CLARKE,

Bo't. of J. C. SMITH & Co.

25 lbs. Sugar,	at \$0.11	\$
5 lbs. Tea,	" .62½	
4 gals. Molasses,	" .49	
30 yds. White Cotton,	" .27	
Received payment,		\$15.93½

J. C. SMITH & Co.
per John Newcomb.

11. Halifax, March 19th, 1866.

William Jones, Esq.,

To W. P. Duffus, Dr.

			$
Jan.	1.	To 15 lbs. Tea, at 50c.	
Dec.	6.	" 25 lbs. Sugar, at 10c.	
Feb.	5.	" 1 bbl. Flour, at $9.50,	
Mar.	14.	" 26 yds. Grey Homespun, at $62\frac{1}{2}$c.	
			$35.75

12. Truro, Feb. 22nd, 1866.

Mr. James Crowe,

Bought of S. Johnson.

		£		
17 lbs. Sugar,	at $6\frac{1}{2}$d.			
$3\frac{1}{2}$ lbs. Tea,	" 2s. $7\frac{1}{2}$d.			
13 lbs. Coffee,	" 1s. 9d.			
3 gals. Burning Fluid,	" 7s. 6d.			
15 lbs. Brown Soap,	" $4\frac{1}{2}$d.			
		£3	9	$3\frac{1}{4}$

13. Pictou, Feb. 17th, 1866—Mr. Andrew Bryden, bought of John Fraser & Co., $17\frac{1}{2}$ yds. superfine cloth at 22s. 6d, per yd., $27\frac{1}{4}$ yds. drab cloth at 12s. 8d., $34\frac{1}{4}$ drugget at 7s. 10d., $18\frac{1}{8}$ yds. broad cloth at 17s. 4d., $29\frac{3}{8}$ yds. serge at 2s. 10d.

Ans. £70 4s. $7\frac{1}{4}$d.

14. Halifax, Feb. 22nd, 1866.—Mr. James Scott, bought of John Young, 24 yds. white cotton, at 27 cents per yard, $17\frac{3}{4}$ yds. flannel at $0.45, $26\frac{1}{2}$ yds. shalloon at $0.37, $5\frac{1}{4}$ yds. broad cloth at $4.75, 15 yds. broad cloth at $1.82, 27 yds. lining cotton at $7\frac{1}{2}$ cents. Ans. 78.53\frac{1}{2}$.

COMPOUND DIVISION.

111. Compound Division is the method of dividing a quantity consisting of several denominations.

112. Compound division is divided into two cases—

1st. When the divisor is an Abstract number.

2nd. When the divisor is a Compound number.

CASE I.

Example.—If 6 acres of land produce 153 bushels 3 pks. 3 qts of oats, how much will 1 acre produce?

OPERATION.

	bus.	pks.	qts.	pts.
6)	153	3	3	0
	25	2	4	1

ANALYSIS.—One acre will produce $\frac{1}{6}$ as much as 6 acres. Writing the divisor on the left of the dividend, we divide 153 bus. by 6, and obtain a quotient of 25 bus., and a remainder of 3 bus. We write the 25 bus. under the denomination of bushels, and reduce the 3 bus. to pecks, making 12 pecks, and the 3 pecks of the dividend added make 15 pecks. Dividing 15 pks. by 6, we obtain a quotient of 2 pks. and a remainder of 3 pks.; writing the 2 pecks under the order of pecks, we next reduce 3 pks to quarts, adding the 3 qts. of the dividend, making 27 qts., which being divided by 6 gives a quotient of 4 qts. and a remainder of 3 qts. Writing the 4 qts. under the order of quarts, and reducing the remainder, 3 qts., to pints, we have 6 pints, which divided by 6 give a quotient of 1 pt., which we write under the order of pints, and the work is finished.

EXAMPLE 2.—When 98 acres produce 2739 bush. 1 pk. 5 qts. of grain, what will 1 acre produce?

OPERATION.

```
    bus.  pks. gal. qts.
98)2739   1    0    5   (27 bus.
   196
   ---
   779
   686
   ---
    93
     4
   ---
   373(3 pks.
   294
   ---
    79
     2
   ---
   158(1 gal.
    98
   ---
    60}
     4}
   ---
   245(2 qts.
   196
   ---
    49
     2
   ---
    98(1 pt.
    98
```

When the divisor is large and not a composite number, we divide by long division, as shown in the operation. From these examples we form the following rule:

Ans. 27 bu. 3 pks. 1 gal. 2 qt. 1 pt.

RULE. **I. Divide the highest denomination, as in simple numbers, and each succeeding denomination in the same manner, if there be no remainder.**

II. If there be a remainder after dividing any denomination, reduce it to the next lower denomination, adding in the given number of that denomination in the dividend, if any, and divide as before.

III. The several partial quotients will be the quotient required.

NOTES.—1. When the divisor is large and is a *composite* number, we may shorten the work by dividing by the factors.

2. When the divisor contains a fraction, as $5\frac{1}{4}$, &c., proceed as directed in Simple Numbers.

Mental Exercises.

1. How much sugar at 9d. per lb. may be bought for 117 pence?

2. How much white sugar at 8d. per lb. may be bought for 1s. 8d.?

3. How much cloth at 7s. per yard, may be bought for £3 17s.?

4. If 9 boxes of figs weighed 28 lbs. 2 oz., what was the weight of 1 box?

5. If 7 bags of rice weighed 12 cwt. 3 qrs. (long weight) what was the weight of 1 box?

6. How much molasses, at $7\frac{1}{2}$d. per quart, may be purchased for £1 17s. 6d.

Exercises for the Slate.

SECTION I.

Answers to be tested as in Reduction ascending.

(1)	£	19	16	0	÷ 2		(11)	£	7947	6	8	÷ 14
(2)		109	1	4	÷ 2		(12)		1640	6	$11\frac{1}{2}$	÷ 14
(3)		324	4	$6\frac{3}{4}$	÷ 3		(13)		2927	2	$4\frac{1}{2}$	÷ 18
(4)		858	10	$11\frac{1}{4}$	÷ 5		(14)		6121	4	7	÷ 20
(5)		904	0	$1\frac{1}{4}$	÷ 5		(15)		4636	3	$0\frac{3}{4}$	÷ 27
(6)		1515	2	3	÷ 6		(16)		21624	4	0	÷ 96
(7)		1513	2	$5\frac{1}{2}$	÷ 7		(17)		25055	6	$4\frac{1}{2}$	÷ 121
(8)		2521	4	6	÷ 8		(18)		48483	12	0	÷ 128
(9)		1488	17	$2\frac{3}{4}$	÷ 11		(19)		80886	13	4	÷ 176
(10)		1624	4	3	÷ 12		(20)		46690	13	0	÷ 216

SECTION II.

In the following exercises the remainders (if any) are divisible by 9.

	tons.	cwt.	qrs.	lbs.	oz.	drs.	(long weight.)
(1)	0	82	0	27	3	8	÷ 45, 81 and 171
(2)		101	0	2	3	11	÷ 54, 63 and 162
(3)	181	2	1	13	15	0	÷ 243, 423 and 432
(4)	1631	18	2	8	10	15	÷ 621, 162 and 261
(5)	72036	1	1	27	10	9	÷ 765, 675 and 999
(6)	80163	0	3	2	0	7	÷ 4302, 5904 and 9045

	lbs.	oz.	dwt.	grs.	
(7)	46	5	11	0	÷ 18, 27 and 36
(8)	326	4	10	9	÷ 126, 261 and 396
(9)	7908	7	2	21	÷ 576, 729 and 891

	lbs.	oz.	drs.	scr.	grs.	
(10)	29	3	0	0	0	÷ 90, 126 and 207
(11)	9876	1	6	1	4	÷ 45, 369 and 639
(12)	305511	0	4	2	8	÷ 702, 837 and 909

	miles.	fur.	po.	yds.	ft.	in.	
(13)	887	3	30	2	0	9	÷ 621, 54 and 702
(14)	2662	3	11	½	2	3	÷ 207, 594 and 945
(15)	4644	3	34	1	0	9	÷ 846, 468 and 711
(16)	59816	1	18	5	0	0	÷ 333, 549 and 27

	dys.	hrs.	min.	sec.	
(17)	1314	0	2	42	÷ 45, 72, 81 and 99
(18)	32626	10	8	24	÷ 612, 711, 549 and 279
(19)	32627	22	4	21	÷ 324, 981, 117 and 819

	yrs.	mo.	wks.	dys.	hrs.	min.	sec.	
(20)	353	0	0	183	6	46	48	÷ 63 and 117
(21)	1278	0	0	199	10	37	12	÷ 972 and 711
(22)	7877	6	0	4	17	34	48	÷ 567 and 756
(23)	3274	1	1	4	10	10	48	÷ 576 and 657

SECTION III.

Take any couplet—as £134 6s. 8½d. and £865 13s. 3½d.—name any number as divisor—say 17—then prefix to the

couplet two numbers whose sum is one less than the divisor chosen—as 7 and 9, and proceed as in the following

OPERATION.

(a)	(b)
£ s. d.	£ s. d.
17)9.134 6 $8\frac{1}{2}$	17)7.865 13 $3\frac{1}{2}$
85 (£537 6 $3\frac{1}{4}$ + $\frac{5}{17}$	68 (£462 13 $8\frac{1}{2}$ + $\frac{13}{17}$
63	106
51	102
124	45
119	34
5	11
20	20
106	233
102	221
4	12
12	12
56	147
51	136
5	11
4	4
22	46
17	34
5	12

ANSWERS.

	£	s.	d.
(a)	537	6	$3\frac{1}{4} + \frac{5}{17}$
(b)	462	13	$8\frac{1}{2} + \frac{12}{17}$
	1000	0	0

The teacher will dictate a list of divisors gradually rising in difficulty. Prefix to the following couplets two numbers whose sum is one less than the divisor chosen, and divide *both* by the divisor, and prove as above.

(LONG WEIGHT.)

	tons.	cwt.	qrs.	lbs.		tons.	cwt.	qrs.	lbs.
(1)	532	15	3	10	and	467	4	0	18
(2)	2372	6	1	21	and	7627	13	2	7
(3)	41632	4	1	13	and	58367	15	2	15
(4)	61824	15	1	16	and	38175	4	2	12

	ac.	ro.	po.	yd.	ft.		ac.	ro.	po.	yd.	ft.
(5)	372	3	20	11	2	and	627	0	19	$18\frac{1}{4}$	7
(6)	2185	1	13	17	2	and	7814	2	26	$12\frac{1}{4}$	1
(7)	34561	1	17	2	6	and	65438	2	22	$27\frac{1}{4}$	3

In the same way exercises may be constructed on all the tables.

CASE II.

113. *When the divisor is a compound number.*

EXAMPLE.—How many times are £5 10s. 10d. contained in £537 10s. 10d.?

OPERATION.

```
£  s.  d.   £   s.  d.
5  10  10)557  10  10(97 times.
20          20
----       -----
110        10750
 12           12
----       ------
1330       129010
           11970
           ------
             9310
             9310
```

ANALYSIS.—Here we reduce both divisor and dividend to pence, that being the lowest denomination contained in either. We then find the divisor, 1330, is contained in the dividend 97 times.

Hence the following

RULE.—Reduce both divisor and dividend to the lowest denomination in either, then proceed as in simple numbers.

SECTION IV.

1. How often is £2 10s. contained in £17 10s.

Ans. 7 times.

2. If a gold ring cost £3 12s. 6d., how many of the same kind may I have for £130 10s.? Ans. 36.

3. How many yards of cloth worth 4s. $6\frac{3}{4}$d. a yard, must be given in exchange for 36 yards at £1 2s. $9\frac{3}{4}$d.? Ans. 180

4. How many barrels are there in 151 bus. 3 pks. 1 gal. of oats, if 1 barrel contain 3 bu. 1 pk. 1 gal.?

Ans. 45 barrels.

SECTION V.

General Exercises.

Divide

1. 69 miles 4 fur. 4 po. 2 yds. by 8.

Ans. 8 m. 5 fur. 20 po. 3 yd.

2. 31 lbs. 11 oz. 15 dwt., by 5. Ans. 6 lb. 4 oz. 15 dwt

3. 35 days 22 h. 52 m. 48 sec., by 6.
Ans. 5 d. 23 h. 48 m. 48 sec.

4. 6429 miles 6 fur. 2 po. 1 yd. 1 ft. 8 in., by 76.
Ans. 84 m. 4 fur. 32 po. 3 yds. 1 ft. 11 in.

5. 646 yds. 3 qrs., by 26. Ans. 24 yds. 3 qrs. 2 nls.

6. £468 3s. $7\frac{1}{2}$d., by $4\frac{1}{2}$. Ans. £104 0s. $9\frac{1}{2}$d. $\frac{6}{9}$.

7. £429 18s. $3\frac{1}{4}$d. by $43\frac{5}{6}$. Ans. £9 16s. $1\frac{3}{4}$d. $\frac{157}{263}$.

8. 8921 tons 15 cwt. 2 qrs. 18 lbs. 15 oz. 15 drs., long weight, by 599. Ans. 14 tons. 17 cwt. 3 qrs. 15 lbs. 9 oz. 9 dr.

9. 7154 days 16 h. 52 m. 48 sec., by 57.
Ans. 125 d. 12 h. 30 m. 24 sec,

10. How often is £5 10s. contained in £38 10s.
Ans. 7 times.

11. How many yards of cloth worth 7s. $8\frac{1}{2}$d. a yard, can be bought for £32 7s. 6d.? Ans. 84 yards.

12. If a single article cost 4s. $6\frac{1}{2}$d., how many dozen may be bought for £196 4s.? Ans. 72.

13. How many yards of cloth worth 4s. $6\frac{3}{4}$d. a yard, must be given in exchange for 36 yards at £1 2s. $9\frac{3}{4}$d. per yard?
Ans. 180.

14. A man travelled by railroad 1000 miles in one day; what was the average rate per hour?
Ans. 41 m. 5 fur. 13 po. 5 ft. 6 in

15. If a family use 10 bbls. of flour in a year, what is the average amount each day? Ans. 5 lb. 5 oz. $14\frac{50}{73}$ dr.

16. A tailor put 276 yds. 3 qrs. of cloth into 20 cloaks; how much cloth did each cloak contain?
Ans. 13 yds. 3 qrs. $1\frac{2}{5}$ nls.

17. A clothier bought 4 pieces of cloth, each containing 60 yds. 2.25 qrs.; after selling $\frac{1}{3}$ of the whole, he had the remainder made into suits containing 9 yd. 2 qr. each; how many suits did it make? Ans. 17.

PROMISCUOUS EXERCISES IN THE PRECEDING RULES.

When going over these and subsequent exercises, the pupil should be required to state in general terms—1st. What is *given* and *what is required* in each problem. 2nd. How it is proposed to do it, giving each step clearly and briefly in its proper order.

If a pupil be thoroughly subjected to this training, day after day at the black-board, clearing up every difficulty in each problem before the teacher and class, his success in arithmetic is in a great measure certain

1. A merchant bought a quantity of sugar for 390 guineas, but paid for it with half-crowns, required how many he gave? Ans. 3276.

2. How many feet will a boy walk to school, which is distant 1 m. 7 fur. 38 po. 4 yds. 2 ft.? Ans. 10541 feet.

3. If $36\frac{1}{2}$ bushels of corn grow on one acre, how many acres will produce 657 bushels? Ans. 18 acres.

4. A man wishes to ship 1560 bushels of shoe pegs in barrels containing 3 bus. 1 pk. each; how many barrels will he require? Ans. 480.

5. A farm consisting of 4 fields, has in one 28 ac. 37 po., in another 27 ac. 2 ro. 26 yds., in another 41 ac. 2 ro. 39 po. 5 ft., and in another 17 ac. 3 ro. 14 yd. 142 inches; required how many inches are in the whole? Ans. 722817646.

6. From the sum of £2 17s. $6\frac{1}{4}$d. + £5 11s. $4\frac{1}{2}$d. + £5 16s. $10\frac{1}{2}$d. + £4 10s. $1\frac{3}{4}$d. + £7 16s. $6\frac{1}{2}$d. take £18 15s. 11d.; multiply the remainder by 11, and divide the product by 13. Ans. £6 12s. $5\frac{1}{2}$d.

7. Reduce 456575 grains to pounds, apothecaries' weight. Ans. 79 ℔. 3 ℥ 1 ʒ 1 ℈ 15 grs.

8. A merchant bought goods for £456 17s. $3\frac{1}{4}$d. and sold them for £530 0s. 6d.; what did he clear on his purchase? Ans. £73 3s. $2\frac{3}{4}$d.

9. Suppose the pulse to beat once in a second, how often will it beat during a year of 365 days? Ans. 31536000 times.

10. A jeweller bought 35 gold watches at £24 10s. each, 49 silver watches at £6 15s. each, 85 gold rings at £1 16s. each, 97 brooches at 17s. 6d. each; how much money did he pay for the whole? Ans. £1426 2s. 6d.

11. Supposing a pair of trousers require 2 yds. 2 qrs. 3 nls.; how much cloth will it require to make 3 doz. pairs? Ans. 96 yds. 3 qrs.

12. What distance will a train travel in 24 hours at the rate of 19 miles 7 fur. 39 po. 5 yds. per hour? Ans. 479 miles 7 fur. 37 po. $4\frac{1}{2}$ yds.

13. A merchant bought 32 tons, 4 cwt. 2 qrs. 14 lbs., short weight, of oats, at 45 cents a bushel; how much money did he pay for the whole? Ans. £213 6s.

14. If seven horses cost £69 6s., what will one cost? Ans. £9 18s.

15. If 3 yds. cost £1 2s. what will 27 yds. cost? Ans. £9 18s.

16. The wages of 8 men amount to £7 6s. $5\frac{1}{2}$d., what will the wages of 128 men amount to? Ans. $456.37

17. If 56 sheep cost $316.80, what will 7 cost?
Ans. $39.60.

18. How long would 36 labourers take to dig a field which 12 men can dig in 27 days? Ans. 9 days.

19. A farmer bought 3 score of lambs at 17s. 6d. each, 2 score of sheep at £1 19s. 11d. each, 24 cows at £9 15s. 8d. each, 6 horses at 39 guineas each, the expenses of getting them all home amonnted to 15 guineas; how much money must he draw from his banker to meet the outlay?
Ans. £628 11s. 8d.

20. If 35 sheep cost $508.90, what is the cost of 5?
Ans. $72.70.

21. When eggs are selling 5 for 2 pence, what should 11 doz. and 3 eggs cost? Ans. 4s. 6d.

22. I went to a shop and bought 7 yds. of cloth at 7s. 6d. per yd., 20 yds. white cotton at 35 cents per yard; what change did I get out of £5? Ans. 18s. $8\frac{3}{4}$d.

23. If 154 bus. 2 pks. 0 qts. cost $173.74, how much will 1 bus. 2 pks. cost? Ans. 1.68\frac{3}{4}$ nearly.

24. An estate consisting of 1977 acres 3 roods is divided into farms containing on an average 98 acres 3 ro. 20 poles each; required the number of farms in the estate?
Ans. 20 farms.

25. If a bushel of barley cost $0.80, what will 21 bus. 2 pks. cost at the same rate? Ans. $17.20.

26. Mr. Flint has two shares in a shoe factory, the capital of which is made up of one hundred and six equal shares, there is a clear gain of $2098.80 at the end of the year. How much should Mr. F. receive? Ans. $39.60.

VULGAR OR COMMON FRACTIONS.

Definitions, Notation and Numeration.

114. If a unit be divided into 2 equal parts, one of these parts is called *one half*.

If a unit be divided into 3 equal parts, one of the parts is called *one third*, two of the parts *two thirds*.

If a unit be divided into 4 equal parts, one of the parts is

called *one fourth*, two of the parts *two fourths*, three of the parts *three fourths*, &c.

The parts are expressed by figures; thus,

One half is written	$\frac{1}{2}$	One fourth is written	$\frac{1}{4}$
One third "	$\frac{1}{3}$	Two fourths "	$\frac{2}{4}$
Two thirds "	$\frac{2}{3}$	Three fourths "	$\frac{3}{4}$

Hence we see that the parts into which a unit is divided take their *name* and their *value* from the *number* of equal parts into which the unit is divided. Thus, if we divide an apple into three equal parts, the parts are called *thirds;* if into 4 equal parts, *fourths*, &c.; and each *fourth* is less in value than each *third*, and the greater the *number* of parts the less the value of each.

When a unit is divided into any number of equal parts, one or more such parts is a fractional part of the whole number, and is called a *fraction.* Hence,

115. A Fraction is one or more of the equal parts of a unit.

116. *To write a fraction* we require two integers, one to express the number of parts into which the whole number is divided, and the other to express the nnmber of parts taken. Thus, if one orange be divided into 5 equal parts, the parts are called *fifths*, and three of these parts are called *three fifths* of an orange.

These may be written

3 *the number of parts taken.*
—
5 *the number of parts into which the orange is divided.*

117. The **Denominator** is the number below the line.
It denominates or names the parts; and
It shows how many parts are equal to a unit.

118. The **Numerator** is the number above the line.
It numerates or numbers the parts; and
It shows how many parts are taken or expressed by the fraction.

119. The **Terms** of a fraction are the numerator and denominator taken together.

120. *Fractions indicate division*, the numerator answering to the dividend, and the denominator to the divisor. Hence,

121. The **Value** of a fraction is the quotient of the numerator divided by the denominator.

Exercises in Notation and Numeration.

Express the following fractions by figures:—

1. Seven *eighths.*
2. Three *twenty-fifths.*
3. Twenty-seven *ninety-sixths.*
4. Seven *one hundred and twenty-sevenths.*
5. Two hundred and four *four hundred and fifty-thirds.*
6. Nine hundred *one thousand and fifty-fourths.*

122. *To analyze* a fraction is to designate and describe its numerator and denominator. Thus, $\frac{3}{4}$ is analyzed as follows:—

4 is the *denominator* and shows that the unit is divided into 4 equal parts; it is the divisor.

3 is the *numerator*, and shows that 3 parts are taken; it is the dividend, or integer divided.

3 and 4 are the terms, considered as dividend and divisor. The value of the fraction is the quotient of $3 \div 4$, or $\frac{3}{4}$.

Read and analyze the following fractions:—

7. $\frac{3}{7}$; $\frac{11}{12}$; $\frac{5}{6}$; $\frac{13}{27}$; $\frac{16}{156}$; $\frac{19}{37}$; $\frac{11}{151}$; $\frac{125}{168}$.
8. $\frac{17}{104}$; $\frac{19}{101}$; $\frac{355}{4367}$; $\frac{51}{1000}$; $\frac{3867}{100017}$.

123. Fractions are distinguished as *Proper* and *Improper.*

A **Proper Fraction** is one whose numerator is less than its denominator. As $\frac{3}{4}$, $\frac{5}{6}$, $\frac{11}{12}$.

An **Improper Fraction** is one whose numerator equals or exceeds its denominator. As $\frac{8}{8}$, $\frac{17}{16}$, $\frac{35}{32}$, $\frac{39}{36}$.

124. A **Mixed Number** is a number expressed by a whole number and a fraction. As $14\frac{1}{2}$, $11\frac{9}{15}$.

125. Since the value of a fraction is the *quotient* obtained by dividing the numerator by the denominator, by the laws of Division we have the following

General principles of Fractions.

126. PRIN. I. **Multiplying the numerator multiplies the fraction, and dividing the numerator divides the fraction.**

PRIN. II. **Multiplying the denominator divides the fraction, and dividing the denominator multiplies the fraction.**

PRIN. III. **Multiplying or dividing both terms of the fraction by the same number does not alter the value of the fraction.**

REDUCTION OF FRACTIONS.

CASE I.

127. *To reduce fractions to their lowest terms.*

A fraction is in its *lowest terms* when its numerator and denominator are prime to each other; that is, when both terms have no common divisor.

EXAMPLE.—Reduce the fraction $\frac{30}{48}$ to its lowest terms.

FIRST OPERATION.

$^{3}\frac{30}{48} = {}^{2}\frac{10}{16} = \frac{5}{8}$ Ans.

ANALYSIS.—Dividing both terms of a fraction by the same number does not alter the value of the fraction or quotient (**126,** Prin. III.,) hence, we divide both terms of $\frac{30}{48}$ by 3, both terms of the result, $\frac{10}{16}$, by 2. As the terms of $\frac{5}{8}$ are prime to each other, the lowest terms of $\frac{30}{48}$ are $\frac{5}{8}$. We have, in effect, cancelled all the factors common to the numerator and denominator.

SECOND OPERATION.

$6)\frac{30}{48} = \frac{5}{8}$, Ans.

In this operation we have divided the terms of the fraction by the greatest common divisor, (**57,**) and thus performed the reduction at a single division. Hence the

RULE. I. Cancel or reject all factors common to both numerator and denominator. Or,

II. Divide both terms by their greatest common measure, or divisor.

Mental Exercises.

Reduce the following fractions to their lowest terms:—

$\frac{6}{8}$; $\frac{3}{9}$; $\frac{18}{26}$; $\frac{21}{27}$; $\frac{18}{36}$; $\frac{5}{55}$; $\frac{9}{54}$; $\frac{8}{72}$; $\frac{16}{72}$; $\frac{26}{78}$; [illegible]/112; $\frac{16}{112}$; $\frac{19}{95}$; $\frac{105}{140}$ and $\frac{112}{126}$.

Exercises for the Slate.

1.	$\frac{155}{180}$	Ans. $\frac{31}{36}$	6.	$\frac{3060}{5940}$	Ans. $\frac{17}{33}$
2.	$\frac{288}{360}$	$\frac{4}{5}$	7.	$\frac{172}{1118}$	$\frac{86}{559}$
3.	$\frac{441}{462}$	$\frac{21}{22}$	8.	$\frac{5643}{5940}$	$\frac{19}{20}$
4.	$\frac{675}{810}$	$\frac{5}{6}$	9.	$\frac{315}{345}$	$\frac{21}{23}$
5.	$\frac{1155}{1260}$	$\frac{11}{12}$	10.	$\frac{684}{1558}$	$\frac{36}{82}$

CASE II.

128. *To reduce an improper fraction to a whole or mixed number.*

EXAMPLE.—Reduce $\frac{32}{7}$ to a whole or mixed number.

OPERATION.

$\frac{32}{7} = 32 \div 7 = 4\frac{4}{7}$, Ans.

ANALYSIS.—Since 7 sevenths equal 1, 32 sevenths are equal to as many times 1 as 7 is contained in 32, which is $4\frac{4}{7}$ times. Hence the following—

RULE.—Divide the numerator by the denominator.

NOTES.—1. When the denominator exactly divides the numerator, the result is a whole number.

2. In all answers containing fractions, the fractions should be reduced to their lowest terms.

Mental Exercises.

1. How many whole things are in 12 halves? 16 halves? 24 halves?
2. How many whole things are in 15 thirds? in 18 thirds?
3. Reduce $\frac{7}{3}$, $\frac{5}{4}$, $\frac{16}{5}$, $\frac{21}{5}$, $\frac{54}{5}$, $\frac{125}{7}$, $\frac{121}{4}$, $\frac{144}{12}$, $\frac{118}{11}$, $\frac{199}{19}$, $\frac{1678}{10}$, to whole or mixed numbers.

Exercises for the Slate.

1. In $\frac{113}{7}$ of a month, how many months? Ans. $16\frac{1}{7}$
2. In $\frac{117}{5}$ of a bushel, how many bushels? $23\frac{2}{5}$
3. In $\frac{563}{3}$ of a dollar, how many dollars? $187\frac{2}{3}$
4. In $\frac{176}{8}$ of a ton, how many tons? 22
5. Reduce $\frac{1487}{701}$ to a mixed number. $2\frac{85}{701}$
6. Reduce $\frac{6570}{292}$ to a mixed number. $22\frac{1}{2}$
7. Change $\frac{2531520}{360}$ to a whole number. 7032

CASE III.

129. *To reduce a whole number to a fraction having a given denominator.*

EXAMPLE.—Reduce 15 bushels to sevenths of a bushel.

OPERATION.

$$\begin{array}{r} 15 \\ 7 \\ \hline \frac{105}{7} \text{ Ans.} \end{array}$$

ANALYSIS.—Since in 1 bushel there are 7 sevenths, in 15 bus. there are 15 times 7 sevenths, which are 105 sevenths $= \frac{105}{7}$. In practice we multiply 15, the number of bushels, by 7, the given denominator, and taking the product 105, for the numerator of a fraction, and the given denominator, 7, for the denominator, we have $\frac{105}{7}$. Hence we have the

RULE. Multiply the whole number by the given denominator, take the product for a numerator, under which write the given denominator.

NOTE.—A whole number is reduced to a fractional form by writing 1 under it for a denominator. Thus $12 = \frac{12}{1}$.

Mental Exercises.

1. Reduce 25 bushels to 4ths of a bushel.
2. Reduce 7 yards to 4ths of a yard.
3. In 56 dollars how many 10ths of a dollar?

4. A man distributed 3 dollars among some poor persons, giving $\frac{1}{5}$ of a dollar to each; how many persons received the money?

Exercises for the Slate.

1. Change 126 to a fraction whose denominator shall be 19. Ans. $\frac{2394}{19}$
2. Reduce 145 pounds to 16ths of a pound. Ans. $\frac{2320}{16}$
3. Change 365 to the form of a fraction.
4. In 196 gallons how many 8ths? Ans. $\frac{1568}{8}$
5. Change 187 to a fraction whose denominator shall be 23. Ans. $\frac{4301}{23}$

CASE IV.

130. *To reduce a mixed number to an improper fraction.*

EXAMPLE.—In $6\frac{1}{8}$ dollars, how many eighths of a dollar?

OPERATION.

$$\begin{array}{r} 6\frac{1}{8} \\ 8 \\ \hline 49 \\ \hline 8 \end{array}$$

ANALYSIS.—Since in 1 dollar there are 8 eighths, in 6 dollars there are 6 times 8 eighths, or 48 eighths, and 48 eighths + 1 eighth = 49 eighths, or $\frac{49}{8}$. From this we derive the following

RULE. Multiply the whole number by the denominator of the fraction; to the product add the numerator, and under the sum write the denominator.

Mental Exercises.

1. How many times $\frac{1}{7}$ are in $6\frac{3}{7}$? in $5\frac{4}{7}$? in $18\frac{3}{7}$? in $16\frac{6}{7}$?
2. How many times $\frac{1}{10}$ are in $5\frac{1}{10}$? in $8\frac{3}{10}$? in $15\frac{4}{10}$? in $22\frac{8}{10}$?
3. In $16\frac{1}{3}$ how many thirds?
4. In $9\frac{7}{12}$ how many twelfths?
5. Reduce $20\frac{2}{3}$ to an improper fraction.
6. How do you change a whole number to a fraction having a required denominator?
7. How do you change a mixed number to an improper fraction?

Exercises for the Slate.

Reduce the following mixed numbers to improper fractions.

1.	$71\frac{3}{5}$	Ans. $\frac{358}{5}$	7.	$225\frac{14}{25}$	Ans. $\frac{5639}{25}$
2.	$161\frac{21}{40}$	$\frac{6461}{40}$	8.	$21\frac{7}{60}$	$\frac{1267}{60}$
3.	$27\frac{19}{31}$	$\frac{856}{31}$	9.	$131\frac{21}{29}$	$\frac{3820}{29}$
4.	$39\frac{13}{38}$	$\frac{1495}{38}$	10.	$156\frac{13}{15}$	$\frac{2353}{15}$
5.	$126\frac{3}{181}$	$\frac{22809}{181}$	11.	$1111\frac{11}{111}$	$\frac{123332}{111}$
6.	$567\frac{4}{121}$	$\frac{68611}{121}$	12.	$1234\frac{123}{124}$	$\frac{153139}{124}$

CASE V.

131. *To reduce a fraction to a given denominator.*

As fractions may be reduced to *lower terms* by division, they may also be reduced to *higher terms* by multiplication; and all the higher terms must be multiples of the lowest terms.

EXAMPLE.–Reduce $\frac{5}{6}$ to a fraction whose denominator is 24.

OPERATION.

$24 \div 6 = 4$

$\frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$

ANALYSIS.—We first divide 24, the required denominator, by 6, the denominator of the given fraction, to ascertain if it be a multiple of this term 6. The division shows that it is a multiple, and that 4 is the factor which must be used to produce this multiple of 6. We therefore multiply both terms of $\frac{5}{6}$ by 4, (**126**, P. III.,) and obtain $\frac{20}{24}$, the desired result. Hence the

RULE.–Divide the required denominater by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.

Mental Exercises.

1. In $\frac{1}{5}$ of 1 how many tenths?
2. In $\frac{3}{4}$ of 1 how many twentieths?
3. In $\frac{7}{9}$ of 1 how many thirty-sixths?
4. In $\frac{5}{7}$ of 1 how many fourteenths?
5. In $\frac{25}{90}$ of 1 how many one hundred and eightieths?

Exercises for the Slate.

1. Reduce $\frac{3}{8}$ to a fraction whose denominator is 264. Ans. $\frac{99}{264}$
2. Reduce $\frac{12}{17}$ to a fraction whose denominator is 51. Ans. $\frac{36}{51}$
3. Reduce $\frac{125}{436}$ to a fraction whose denominator is 3488. Ans. $\frac{1000}{3488}$
4. Reduce $\frac{5}{9}$ to a fraction whose denominator is 6300. Ans. $\frac{3500}{6300}$

CASE VI.

132. *To reduce two or more fractions to a common denominator.*

A **Common Denominator** is a denominator common to two or more fractions. Thus 4 is the common denominator of $\frac{1}{4}$, $\frac{3}{4}$ and $\frac{2}{4}$.

EXAMPLE.—Reduce $\frac{3}{4}$ and $\frac{5}{6}$ to a common denominator.

OPERATION.

$\frac{3}{4} \times \frac{6}{6} = \frac{18}{24}$

$\frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$

ANALYSIS.—We multiply the terms of the first fraction by the denominator of the second, and the terms of the second fraction

by the denominator of the first, (**126**.) This must reduce each fraction to the same denominator, for each new denominator will be the product of the given denominators. Hence the

RULE. Multiply the terms of each fraction by the denominators of all the other fractions.

NOTE.—Mixed numbers must first be reduced to improper fractions.

Exercises for the Slate.

Reduce to equivalent fractions having a common denominator.

1. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{1}{9}$. Ans. $\frac{316}{432}$, $\frac{324}{432}$, $\frac{360}{432}$, $\frac{48}{432}$
2. $\frac{4}{5}$, $\frac{7}{12}$, and $\frac{5}{6}$. $\frac{288}{360}$, $\frac{210}{360}$, $\frac{300}{360}$
3. $\frac{9}{16}$, $\frac{1}{3}$ and $\frac{2}{9}$. $\frac{243}{432}$, $\frac{144}{432}$, $\frac{96}{432}$
4. $\frac{5}{6}$, $2\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{3}$. $\frac{120}{144}$, $\frac{360}{144}$, $\frac{108}{144}$, $\frac{48}{144}$
5. $1\frac{7}{8}$, $\frac{3}{10}$ and 4. $\frac{150}{80}$, $\frac{24}{80}$, $\frac{320}{80}$

CASE VII.

133. *To reduce fractions to the least common denominator.*

The **Least Common Denominator** of two or more fractions, is the least common denominator to which they can all be reduced, and it must be the least common multiple of the lowest denominators.

EXAMPLE.—Reduce $\frac{1}{6}$, $\frac{3}{4}$ and $\frac{5}{8}$ to the least common denominator.

OPERATION.

$$\begin{array}{r|ccc} 2 & 6 & 4 & 8 \\ \hline & 3 & 2 & 4 \end{array}$$

$3 \times 4 \times 2 = 24$

OR,

$6 = 2 \times 3$
$4 = 2 \times 2$
$8 = 2 \times 2 \times 2$

Therefore $2 \times 2 \times 2 \times 3 = 24$

Since $24 \div 6 = 4 \therefore \frac{1}{6} \times \frac{4}{4} = \frac{4}{24}$
" $24 \div 4 = 6 \therefore \frac{3}{4} \times \frac{6}{6} = \frac{18}{24}$
" $24 \div 8 = 3 \therefore \frac{5}{8} \times \frac{3}{3} \div \frac{15}{24}$

ANALYSIS.—We find the least common multiple of the given denominators, which is 24.—This must be the least common denominator to which the fractions can be reduced. We then divide this least common multiple, 24, by the denominator of the given fraction, and multiplying each term of that fraction by the quotient, (**126**,) we have the answer.—

Hence the

RULE. I. Find the least common multiple of the given denominators, for the least common denominator

II. Divide this common denominator by each of the given denominators, and multiply each numerator by the corresponding quotient. The products will be the new numerators.

NOTE. 1. Mixed numbers must first be reduced to improper fractions.

2. If the several fractions are not in their lowest terms, they should be reduced to their lowest terms before applying the rule.

Exercises for the Slate.

Reduce the following to their least common denominator.

1. $\frac{2}{25}$, $\frac{3}{10}$, $\frac{47}{50}$ and $\frac{4}{75}$. Ans. $\frac{12}{150}$, $\frac{45}{150}$, $\frac{141}{150}$, $\frac{8}{150}$
2. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{20}$, and $\frac{11}{12}$. Ans. $\frac{60}{120}$, $\frac{90}{120}$, $\frac{100}{120}$, $\frac{105}{120}$, $\frac{54}{120}$, $\frac{110}{120}$
3. $\frac{1}{2}$, $\frac{4}{7}$, $\frac{3}{16}$, and $\frac{2}{21}$. $\frac{168}{336}$, $\frac{192}{336}$, $\frac{63}{336}$, $\frac{32}{336}$
4. $\frac{3}{7}$, $\frac{9}{14}$, $\frac{11}{28}$ and $5\frac{3}{7}$. $\frac{12}{28}$, $\frac{18}{28}$, $\frac{11}{28}$, $\frac{152}{28}$
5. $\frac{4}{9}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{12}$. $\frac{16}{36}$, $\frac{24}{36}$, $\frac{12}{36}$, $\frac{9}{36}$, $\frac{6}{36}$, $\frac{3}{36}$
6. $7\frac{3}{4}$, $5\frac{6}{11}$, 7, and 8. $\frac{341}{44}$, $\frac{244}{44}$, $\frac{308}{44}$, $\frac{352}{44}$
7. $\frac{25}{40}$, $\frac{25}{120}$, and $\frac{14}{64}$. $\frac{60}{96}$, $\frac{20}{96}$, $\frac{21}{96}$
8. $\frac{4}{15}$, $\frac{5}{75}$, $\frac{32}{56}$, and $4\frac{1}{3}$. $\frac{28}{105}$, $\frac{7}{105}$, $\frac{60}{105}$, $\frac{455}{105}$
9. $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$, $5\frac{1}{6}$, and $\frac{7}{9}$. $\frac{54}{36}$, $\frac{84}{36}$, $\frac{117}{36}$, $\frac{186}{36}$, $\frac{28}{36}$
10. $\frac{4}{11}$, $7\frac{1}{2}$, $\frac{20}{33}$ and 5. $\frac{24}{66}$, $\frac{495}{66}$, $\frac{40}{66}$, $\frac{330}{66}$

ADDITION OF FRACTIONS.

CASE I.

134. *To add fractions having a common denominator.*

EXAMPLE.—What is the sum of $\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$ and $\frac{7}{9}$?

OPERATION.

$\frac{1}{9}+\frac{2}{9}+\frac{3}{9}+\frac{7}{9}=\frac{13}{9}=1\frac{4}{9}$, Ans.

ANALYSIS.—Since the given fractions have a common denominator, 9, their sum may be found by adding their numerators, 1, 2, 3, and 7, and placing the sum, 13, over the common denominator. We thus obtain $\frac{13}{9}=1\frac{4}{9}$, the required sum. Hence the

RULE. Add the numerators, and place the sum over the common denominator.

NOTE.—If the amount be an improper fraction, reduce it to a whole or a mixed number.

Exercises for the Slate.

1. Add $\frac{3}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$ and $\frac{9}{10}$. Ans. $3\frac{4}{5}$
2. Add $\frac{5}{12}$, $\frac{8}{12}$, $\frac{4}{12}$, $\frac{7}{12}$, and $\frac{11}{12}$. $2\frac{1}{2}$
3. Add $\frac{1}{20}$, $\frac{3}{20}$, $\frac{7}{20}$, $\frac{9}{20}$, $\frac{11}{20}$ and $\frac{17}{20}$. $2\frac{2}{5}$
4. Find the sum of $\frac{5}{24}$, $\frac{7}{24}$, $\frac{11}{24}$ and $\frac{21}{24}$. $1\frac{5}{6}$
5. Find the sum of $\frac{13}{225}$, $\frac{76}{225}$, $\frac{101}{225}$ and $\frac{125}{225}$. $1\frac{2}{5}$

CASE II.

135. *To add fractions having different denominators.*

EXAMPLE.—What is the sum of $\frac{4}{5}$ and $\frac{7}{9}$?

FIRST OPERATION.

$\frac{4}{5} + \frac{7}{9} = \frac{36}{45} + \frac{35}{45} = \frac{71}{45} = 1\frac{26}{45}$ Ans.

ANALYSIS.—In whole numbers we can add like numbers only, or those of the same unit value; so in fractions we can add the numerators when they have a common denominator, but not otherwise. As $\frac{4}{5}$ and $\frac{7}{9}$ have not a common denominator, we first reduce them to a common denominator, **(132 or 133)** and then add the numerators, $36 + 35 = 71$, the same as whole numbers, and place the sum over the common denominator.

SECOND OPERATION.

$$\left.\begin{array}{l}\frac{4}{5} = 36 \\ \frac{7}{9} = 35\end{array}\right\} 45 \text{ L. C. M.}$$

$$\frac{71}{45} = 1\frac{26}{45} \text{ Ans.}$$

ANALYSIS.—Since it is easier to perform addition when the numbers are in columns, we therefore place the new numerators as in addition of simple numbers and write the common denominator at the side. From the above examples we have the following

RULE. I. Reduce the fractions to a common or to their least common denominator.

II. Add the numerators, and place the sum over the common denominator.

NOTE.—If the amount be an improper fraction, reduce it to a whole or a mixed number.

Exercises for the Slate.

1. Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$ and $\frac{9}{10}$. Ans. $3\frac{103}{120}$
2. Add $\frac{3}{4}$, $\frac{1}{8}$, $\frac{2}{7}$ and $\frac{5}{12}$. $1\frac{97}{168}$
3. Add $\frac{42}{140}$, $\frac{9}{70}$, $\frac{7}{28}$ and $\frac{1}{14}$. $\frac{3}{4}$
4. Add $\frac{7}{8}$, $\frac{11}{12}$, $\frac{17}{18}$, $\frac{23}{24}$ and $\frac{26}{27}$. $4\frac{71}{108}$
5. Add $\frac{8}{9}$, $\frac{9}{10}$, $\frac{10}{11}$, $\frac{11}{12}$, $\frac{12}{13}$, $\frac{13}{14}$ and $\frac{14}{15}$. $6\frac{14401}{360360}$

CASE III.

136. *To add mixed numbers*

EXAMPLE.—Add $3\frac{1}{2}$, $5\frac{3}{4}$, and $7\frac{1}{16}$.

OPERATION.

$$\left.\begin{array}{l} 3\frac{1}{2} = 8 \\ 5\frac{3}{4} = 12 \\ 7\frac{1}{16} = 1 \end{array}\right\} \text{16 L. C. M. or C. D.}$$

$$\begin{array}{ll} 15 & \frac{21}{16} = 1\frac{5}{16}. \\ 1\frac{5}{16} & \\ \hline 16\frac{5}{16} \text{ Ans.} & \end{array}$$

ANALYSIS.—The sum of the fractions, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{16}$, is $1\frac{5}{16}$; the sum of the integers 3, 5, and 7, is 15: and the sum of both fractions and integers is $16\frac{5}{16}$. Hence the following—

RULE.—Add the fractions and integers separately, and then add their sums.

NOTE.—When the mixed numbers are small, they may be reduced to improper fractions, and then added by the usual method.

Exercises for the Slate.

1. Add $5\frac{1}{2}$ $3\frac{1}{3}$, $4\frac{5}{8}$ and $6\frac{1}{4}$. Ans. $19\frac{17}{24}$
2. Find the sum of $\frac{7}{8}$, $1\frac{7}{12}$, $10\frac{5}{6}$, and 5. $18\frac{7}{24}$
3. Find the sum of $126\frac{1}{4}$, $183\frac{3}{8}$, and $196\frac{3}{16}$. $505\frac{13}{16}$
4. What is the sum of $3\frac{1}{7}$, $126\frac{1}{8}$, and $144\frac{5}{28}$. $273\frac{25}{56}$
5. Bought 5 lots of land containing $12\frac{7}{8}$ acres, $105\frac{9}{10}$ acres, $18\frac{1}{4}$ acres, $15\frac{11}{12}$ acres, and $5\frac{1}{6}$ acres; how many acres are in the 5 lots? Ans. $158\frac{13}{120}$
6. A grain merchant bought $126\frac{3}{4}$ bushels of wheat for $136\frac{9}{10}$ dollars, $367\frac{1}{4}$ bushels of barley for $219\frac{3}{4}$ dollars, $506\frac{11}{12}$ bushels of oats for $236\frac{3}{16}$ dollars; how many bnshels of grain did he buy, and how much did he pay for the whole?

$$\text{Ans.} \left\{\begin{array}{l} 1000\frac{11}{12} \text{ bushels.} \\ 592\frac{67}{80} \text{ dollars.} \end{array}\right.$$

SUBTRACTION OF FRACTIONS.

CASE I.

137. *To subtract fractions having a common denominator.*

EXAMPLE.—From $\frac{7}{10}$ take $\frac{3}{10}$.

OPERATION.

$$\frac{7}{10} - \frac{3}{10} = \frac{7-3}{10} = \frac{4}{10} = \frac{2}{5}$$

ANALYSIS.— Since the given fractions have a common denominator, 10, we find

the difference by subtracting 3, the less numerator, from 7, the greater, and write the remainder, 4, over the common denominator, 10. We thus obtain $\frac{4}{10} = \frac{2}{5}$, the required difference. Hence the following—

RULE. **Subtract the numerator of the subtrahend from the numerator of the minuend, and place the difference over the common denominator.**

Exercises for the Slate.

1.	From $\frac{5}{9}$ take $\frac{3}{9}$.	Ans. $\frac{2}{9}$
2.	From $\frac{6}{13}$ take $\frac{5}{13}$.	$\frac{1}{13}$
3.	From $\frac{15}{11}$ take $\frac{8}{11}$.	$\frac{7}{11}$
4.	From $\frac{63}{163}$ take $\frac{54}{163}$.	$\frac{9}{163}$
5.	From $\frac{75}{196}$ take $\frac{47}{196}$.	$\frac{1}{7}$
6.	From $\frac{182}{348}$ take $\frac{110}{348}$.	$\frac{6}{29}$

CASE II.

138. *To subtract fractions having different denominators.*

EXAMPLE.—From $\frac{5}{8}$ take $\frac{3}{7}$.

OPERATION.

$$\frac{5}{8} - \frac{3}{7} = \frac{35}{56} - \frac{24}{56} = \frac{35-24}{56} = \frac{11}{56}, \text{ Ans.}$$

OR,

$$\left.\begin{array}{l}\frac{5}{8} = 35 \\ \frac{3}{7} = 24\end{array}\right\} 56 \text{ C. D.}$$

$\frac{11}{56}$, Ans.

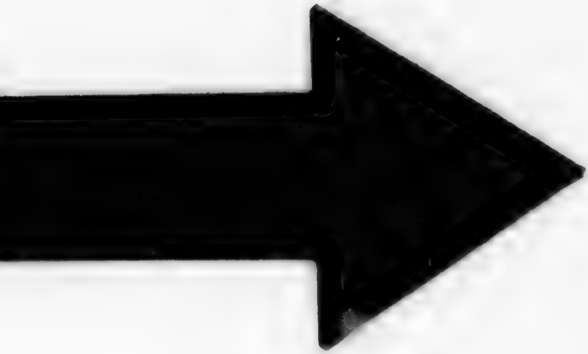
ANALYSIS.—As in whole numbers we subtract *like* numbers only, or those having the same unit value, so, we can subtract fractions only when they have a common denominator. As $\frac{5}{8}$ and $\frac{3}{7}$ have not a common denominator, we first reduce them to a common denominator, and then subtract the less numerator, 24, from the greater numerator, 35, and write the difference, 11, over the common denominator, 56. We thus obtain $\frac{11}{56}$, the required difference. Hence the following—

RULE. **Reduce the fractions to a common denominator and subtract as in the former rule.**

Exercises for the Slate.

1.	From $\frac{7}{8}$ take $\frac{5}{6}$.	Ans. $\frac{1}{24}$
2.	From $\frac{16}{31}$ take $\frac{5}{62}$.	$\frac{27}{62}$
3.	From $\frac{84}{120}$ take $\frac{4}{35}$.	$\frac{41}{70}$

[redacted]



4. From $\frac{35}{80}$ take $\frac{14}{200}$. $\frac{147}{400}$

5. From $\frac{18}{56}$ take $\frac{81}{196}$. $\frac{29}{392}$

CASE III.

139, *To subtract mixed numbers.*

EXAMPLE.—What is the difference between $18\frac{1}{4}$ and $7\frac{1}{3}$.

OPERATION.

$$18\frac{1}{4} = 18\frac{3}{12}$$
$$7\frac{1}{3} = 7\frac{4}{12}$$
$$\overline{10\frac{11}{12}}$$

OR,

$$\left.\begin{array}{l} 18\frac{1}{4} = 3 \\ 7\frac{1}{3} = 4 \end{array}\right\} 12 \text{ C. D.}$$
$$\overline{10} \quad \overline{\tfrac{11}{12}}$$

ANALYSIS.—We first reduce the fractional parts, $\frac{1}{4}$ and $\frac{1}{3}$, to a common denominator, 12. Since we cannot take $\frac{4}{12}$ from $\frac{3}{12}$, we add $1 = \frac{12}{12}$ to $\frac{3}{12}$, which makes $\frac{15}{12}$, and $\frac{4}{12}$ from $\frac{15}{12}$ leaves $\frac{11}{12}$. Again, having added 1 to the upper number, we must add 1 to the lower number, so that the difference between the two numbers may not be altered; and adding 1 to 7 we have 8, which taken from 18, leaves 10. We thus obtain $10\frac{11}{12}$ the difference required.—Hence the following—

RULE.—Reduce the fractional parts to a common denominator, and then subtract the fractional and integral parts separately. Or,

We may reduce the mixed numbers to improper fractions, and subtract the less from the greater by the usual method.

Exercises for the Slate.

1. From $8\frac{1}{4}$ take $5\frac{1}{8}$. Ans. $3\frac{1}{8}$
2. From $27\frac{5}{6}$ take $19\frac{7}{10}$. $8\frac{2}{15}$
3. From $5\frac{1}{2}$ take $4\frac{3}{4}$. $\frac{3}{4}$
4. From 27 take $18\frac{1}{6}$. $8\frac{5}{6}$
5. From $3\frac{17}{30}$ take $1\frac{48}{125}$. $2\frac{137}{750}$
6. From a barrel of Kerosene oil containing $56\frac{1}{8}$ gallons $27\frac{1}{4}$ gallons were drawn; how many gallons remained? Ans. $28\frac{7}{8}$
7. If flour, which cost $\$6\frac{7}{8}$ per barrel, be sold for $\$7\frac{3}{4}$ per barrel, what will be the gain per barrel? Ans. $\$\frac{7}{8}$
8. From the sum of $5\frac{1}{4}$, $3\frac{1}{8}$ and $8\frac{1}{16}$ take the sum of $2\frac{1}{8}$, $7\frac{7}{9}$ and $\frac{13}{27}$. Ans. $6\frac{23}{432}$
9. What fraction added to $\frac{13}{14}$ will make $\frac{19}{20}$? Ans. $\frac{3}{140}$
10. A man having $368\frac{1}{8}$ dollars, paid $\$100\frac{7}{10}$ for a horse, $\$25\frac{1}{4}$ for a set of harness, $\$\frac{3}{16}$ for a whip, and $\$175\frac{7}{12}$ for a waggon; how much had he left? Ans. $\$66\frac{97}{240}$

MULTIPLICATION OF FRACTIONS.

CASE I.

140. *To multiply a fraction by an integer.*

EXAMPLE 1.—If 1 yard of cloth cost £$\frac{3}{4}$, how much will 7 yds. cost?

OPERATION

$\frac{3}{4} \times 7 = \frac{21}{4} = 5\frac{1}{4}$ Ans.

ANALYSIS.—Since 1 yd. cost 3 *fourths* of one pound, 7 yds. will cost 7 times 3 *fourths* of one pound, or 21 *fourths*, equal to £$5\frac{1}{4}$.

A fraction is multiplied by multiplying its numerator, **(126.)**

EXAMPLE 2.—If 1 pound of Tea cost $\frac{9}{20}$ of a dollar, how much will 4 lbs. cost?

OPERATION.

$$\frac{9}{\underset{5}{\cancel{20}}} \times \cancel{4} = \frac{9}{5} = 1\frac{4}{5} \text{ Ans.}$$

ANALYSIS.—Since 4, the multiplier, is a factor of 20, the denominator, of the multiplicand, we perform the multiplication by dividing the denominator, 20, by the multiplier, 4, and we have $\frac{9}{5} = 1\frac{4}{5}$ dollars.

A fraction is multiplied by dividing its denominator, **(126)**. Hence the following—

RULE. Multiply the numerator of the fraction by the whole number, and write the product over the numerator. Or,

Divide the denominator by the whole number, when this can be done without a remainder.

Exercises for the Slate.

1. Multiply $\frac{5}{8}$ by 6. Ans. $3\frac{3}{4}$
2. Multiply $\frac{11}{12}$ by 9. $8\frac{1}{4}$
3. Multiply $\frac{8}{15}$ by 5. $2\frac{2}{3}$
4. Multiply $\frac{4}{21}$ by 84. 16
5. Multiply $\frac{7}{25}$ by 55. $15\frac{2}{5}$
6. Multiply $6\frac{1}{4}$ by 7. $43\frac{3}{4}$

OPERATION.

$$\begin{array}{r} 6\frac{1}{4} \\ 7 \\ \hline 1\frac{3}{4} \\ 42 \\ \hline 43\frac{3}{4} \end{array}$$

or,

$6\frac{1}{4} = \frac{25}{4}$

$\frac{25}{4} \times 7 = \frac{175}{4} = 43\frac{3}{4}$

ANALYSIS.—In multiplying a mixed number, we first multiply the fractional part, and then the integer, and add the two products, or we reduce the mixed number to an improper fraction, and then multiply it.

7. Multiply $17\frac{1}{8}$ by 5. Ans. $85\frac{5}{8}$
8. Multiply $\frac{31}{121}$ by 7. $1\frac{96}{121}$
9. Multiply $16\frac{5}{8}$ by 16. 266
10. Multiply $\frac{101}{136}$ by 544. 404
11. If 1 ton of hay cost $\$8\frac{8}{10}$, what will 12 tons cost? Ans. $\$105\frac{3}{5}$
12. What will 14 yds. of silk cost at $1\frac{7}{8}$ dollars per yard? Ans. $\$26\frac{1}{4}$

CASE II.

141. *To multiply a whole number by a fraction.*

EXAMPLE.—At 83 dollars an acre, how much will $\frac{3}{5}$ of an acre cost?

OPERATION.

83 price of 1 acre.
3
—
5)249 = cost of 3 acres.
—
$49\frac{4}{5}$ = " $\frac{3}{5}$ of an acre.

ANALYSIS.—Multiplying the price of 1 acre by 3, we have the price of 3 acres; and as $\frac{1}{5}$ of 3 acres is the same as $\frac{3}{5}$ of 1 acre, we divide the cost of 3 acres by 5, and we have the cost of $\frac{3}{5}$ of an acre.—

Hence the following—

RULE. Multiply the given number by the numerator, and divide the product by the denominator.

NOTE.—When the denominator is exactly contained in the given number, it will be found easier to first divide by it, and then multiply the quotient by the numerator.

Exercises for the Slate.

1. Multiply 4 by $\frac{2}{3}$. Ans. $2\frac{2}{3}$
2. Multiply 165 by $\frac{4}{33}$. 20
3. Multiply 457 by $\frac{7}{12}$. $266\frac{7}{12}$
4. What is $\frac{11}{128}$ of 4261. $366\frac{23}{128}$
5. What is $\frac{7}{12}$ of 1644. 959
6. Multiply 26 by $5\frac{3}{8}$.

OPERATION.

26
$5\frac{3}{8}$
—
$9\frac{3}{4} = \frac{3}{8}$ of 26
120
—
$139\frac{3}{4}$, Ans.

Or $5\frac{3}{8} = \frac{43}{8}$
$26 \times \frac{43}{8} = \frac{1118}{8}$
$139\frac{3}{4}$ Ans.

ANALYSIS.—We multiply by the integer and fraction separately, and add the products; or reduce the mixed number to an improper fraction, and then multiply by it.

7. Multiply 83 by $7\frac{1}{5}$. Ans. $597\frac{3}{5}$
8. Multiply 45 by $8\frac{1}{3}$. 375
9. Multiply 156 by $\frac{27}{39}$. 108
10. If a man walk 16 miles in one day, how many will he travel in $112\frac{3}{8}$ days? Ans. 1798
11. At 18 dollars per ton, what is the cost of $18\frac{7}{9}$ tons of hay? Ans. $338

CASE III.

142. *To multiply a fraction by a fraction.*

EXAMPLE 1.—At $\frac{3}{8}$ of a dollar per yard, how much will $\frac{3}{4}$ of a yard cost?

OPERATION.

$\frac{3}{8} \times \frac{1}{4} = \frac{3}{32} \times 3 = \frac{9}{32}$ Ans.

ANALYSIS.—Since 1 yard cost $\frac{3}{8}$ of a dollar, $\frac{1}{4}$ of a yard will cost $\frac{1}{4}$ of $\frac{3}{8}$, which is $\frac{3}{32}$ of a dollar; and as $\frac{1}{4}$ of a yard costs $\frac{3}{32}$ of a dollar, $\frac{3}{4}$ of a yard will cost 3 times as much, or $\frac{3}{32} \times 3 = \frac{9}{32}$. It will readily be seen that we have multiplied together the two numerators, 3 and 3, for a new numerator, and the two denominators, 8 and 4, for a new denominator, as shown in the whole work of the operation. Hence for multiplication of fractions we have this general

RULE. I. Reduce all integers and mixed numbers to improper fractions.

II. Multiply together the numerators for a new numerator, and the denominators for a new denominator.

NOTE.—Cancel all factors common to numerators and denominators.

EXAMPLE 2.—Multiply $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{8}{9}$.

OPERATION.

$$\frac{\cancel{3}}{4} \times \frac{5}{\underset{2}{\cancel{6}}} \times \frac{7}{\cancel{8}} \times \frac{\cancel{8}}{9} = \frac{35}{72} \text{ Ans.}$$

OR

$$\begin{array}{r|l} 4 & \cancel{3} \\ 2\ \cancel{6} & 5 \\ \cancel{8} & 7 \\ 9 & \cancel{8} \\ \hline 72 & 35 = \frac{35}{72}, \text{ Ans.} \end{array}$$

NOTE.—Fractions with the word *of* between them are sometimes called *compound fractions*. The word *of* is simply an equivalent for the sign ($\times$) of multiplication, and signifies that the numbers between which it is placed are to be multiplied together.

Exercises for the Slate.

1. Multiply $\frac{2}{3}$ by $\frac{3}{4}$. Ans. $\frac{6}{12} = \frac{1}{2}$
2. Multiply $\frac{5}{8}$ by $\frac{7}{10}$. $\frac{7}{16}$
3. Multiply $\frac{68}{95}$ by $\frac{100}{250}$. $\frac{136}{475}$
4. Multiply $\frac{1}{2}$ of 75 by $\frac{2}{3}$ of 28. 700
5. Multiply $\frac{4}{5}$ of $10\frac{3}{4}$ by $\frac{2}{3}$ of $8\frac{1}{4}$. $47\frac{3}{10}$
6. Multiply $\frac{7}{8}$ of $\frac{9}{10}$ of 20 by $25\frac{1}{2}$. $401\frac{5}{8}$
7. At $\frac{8}{9}$ of a dollar per pound, what will $\frac{3}{8}$ of a pound cost? Ans. $\frac{1}{3}$ of a doll.
8. What cost $125\frac{1}{2}$ bbls. of flour at $\$7\frac{3}{4}$ per bbl.? Ans. $\$972\frac{5}{8}$
9. If a man travels $40\frac{3}{5}$ miles per day, how far will he travel in $135\frac{1}{2}$ days? Ans. $5501\frac{3}{10}$ miles.
10. Bought $126\frac{1}{4}$ barrels of flour at $\$7\frac{3}{8}$ per barrel; and sold $58\frac{1}{2}$ barrels at $\$7\frac{5}{8}$ per barrel, and the balance at $\$8\frac{1}{16}$ per barrel; how much was the gain? Ans. $\$61\frac{13}{64}$

DIVISION OF FRACTIONS.

CASE I.

143. *To divide a fraction by a whole number.*

EXAMPLE.—If 4 yards of cotton cost $\frac{8}{9}$ of a dollar, what will 1 yard cost?

OPERATION.

$\frac{8}{9} \div 4 = \frac{2}{9}$. Ans.

ANALYSIS.—If 4 yards cost $\$\frac{8}{9}$, 1 yard will cost 1 *fourth* of $\frac{8}{9}$, or $\frac{8}{9}$ divided by 4. Since a fraction is divided by dividing its numerator (**126**), we divide the numerator of the fraction, $\frac{8}{9}$, by 4, and we have $\frac{2}{9}$, the answer.

EXAMPLE 2.—If 5 bushels of apples cost $\frac{11}{12}$ of a pound, what will 1 bushel cost?

OPERATION.

$\frac{11}{12} \div 5 = \frac{11}{12 \times 5} = \frac{11}{60}$, Ans.

ANALYSIS.—Here we cannot divide the numerator by 5 without leaving a remainder; but since a fraction is divided by multiplying the denominator, (**126**), we multiply the denominator of the fraction, $\frac{11}{12}$, by 5, and we have $\frac{11}{60}$, the required result. Hence the following—

RULE. Divide the numerator by the whole number, when it can be done without leaving a remainder; but when this cannot be done, multiply the denominator by the whole number.

Exercises for the Slate.

1. Divide $\frac{18}{23}$ by 9. Ans. $\frac{2}{23}$
2. Divide $\frac{24}{31}$ by 8. $\frac{3}{31}$
3. Divide $\frac{75}{125}$ by 25. $\frac{3}{125}$
4. Divide $\frac{64}{121}$ by 16. $\frac{4}{121}$
5. Divide $\frac{13}{17}$ by 14. $\frac{13}{238}$
6. Divide $\frac{51}{72}$ by 6. $\frac{51}{432}$
7. At 18 dollars per ton, what part of a ton of hay can be bought for $\$\frac{7}{8}$? Ans. $\frac{7}{144}$
8. If 9 bushels of oats cost $7\frac{1}{8}$ dollars, how much will 1 bushel cost?

OPERATION.

$$7\tfrac{1}{8} = \tfrac{57}{8}$$
$$\tfrac{57}{8} \div 9 = \tfrac{57}{72} = \tfrac{19}{24}, \text{ Ans.}$$

NOTE.—We reduce the mixed number to an improper fraction and divide as before.

9. If 8 barrels of flour cost $126\frac{5}{8}$ dollars, how much will 1 barrel cost?

OPERATION.

$$\begin{array}{r} 8)126\frac{5}{8} \\ \hline 15\frac{53}{64} \end{array}$$

ANALYSIS.—Here we first divide as in simple numbers, and we have a remainder of $6\frac{5}{8}$. We reduce this to an improper fraction, $\frac{53}{8}$, which we divide (as in Ex. 1) and annex the result, $\frac{53}{64}$, to the partial quotient, 15, and we have, $15\frac{53}{64}$, the required result.

10. If $126\frac{3}{8}$ dollars were paid for 4 cows, what was the price of each? Ans. $31\frac{19}{32}$
11. If 22 horses eat $\frac{1}{8}$ of $1126\frac{1}{8}$ pounds of hay in a day, how much does each horse consume? Ans. $6\frac{561}{1408}$ pounds.

CASE II.

144. *To divide a whole number by a fraction.*

EXAMPLE.—How many pounds of tea at $\frac{3}{4}$ of a dollar can be purchased for 15 dollars?

FIRST OPERATION.

$$\begin{array}{r} 15 \\ 4 \\ \hline 3)60 \\ \hline 20 \text{ lbs. Ans.} \end{array}$$

ANALYSIS.—As many pounds as $\frac{3}{4}$ of a dollar, the price of 1 pound is contained times in 15 dollars. Whole numbers cannot be divided by *fourths*, because they are not of the same denomination. Reducing 15 dollars to *fourths* by multiplying by 4, we have 60 *fourths*; and 3 *fourths* is contained in 60 *fourths* 20 times, the required number of pounds.

SECOND OPERATION.

$$\begin{array}{r} 3)15 \\ \hline 5 \\ 4 \\ \hline 20 \text{ pounds.} \end{array}$$

ANALYSIS.—Here we divide the integer by the numerator of the fraction, and multiply the quotient by the denominator, which produces the same result. Hence the following—

RULE. Multiply by the denominator and divide the product by the numerator.

Exercises for the Slate.

1. Divide 21 by $\frac{3}{7}$. Ans. 49
2. Divide 63 by $\frac{9}{11}$. 77
3. Divide 316 by $\frac{9}{25}$. $877\frac{7}{9}$
4. Divide 75 by $\frac{5}{9}$. 135
5. Divide 120 by $10\frac{3}{4}$. $11\frac{7}{43}$
6. Divide 145 by $12\frac{1}{6}$. $11\frac{67}{73}$
7. Divide $\frac{5}{8}$ of 320 by $\frac{5}{6}$ of $9\frac{1}{3}$. $25\frac{5}{7}$
8. Divide $\frac{1}{4}$ of \$32 by $\frac{1}{3}$ of $7\frac{1}{2}$. $\$3\frac{1}{5}$

CASE III.

145. *To divide a fraciton by a fraction.*

EXAMPLE.—At $\frac{2}{3}$ of a dollar per pound, how much tea can be bought for $\frac{4}{5}$ of a dollar?

OPERATION.

$\frac{4}{5} \times 3 = \frac{12}{5}$

$\frac{12}{5} \div 2 = \frac{12}{10} = 1\frac{1}{5}$ Ans.

ANALYSIS.—As many pounds as $\frac{2}{3}$ of a dollar is contained in $\frac{4}{5}$ of a dollar. 1 is contained in $\frac{4}{5}$, $\frac{4}{5}$ times, and $\frac{1}{3}$ is contained 3 times as many times as 1, or 3 times $\frac{4}{5}$, which is $\frac{12}{5}$ times, which is the number of pounds that can be bought at $\frac{1}{3}$ of a dollar per pound; but $\frac{2}{3}$ is contained by $\frac{1}{2}$ as many times as $\frac{1}{3}$, and $\frac{12}{5}$ divided by 2 gives $\frac{12}{10}$, equal to $1\frac{1}{5}$ times, or the number of pounds that can be bought at $\frac{2}{3}$ of a dollar per pound.

We see in the operation that we have multiplied the dividend by the denominator of the divisor, and divided the result by the numerator of the divisor. Hence for division of fractions we have this general

RULE. I. Reduce whole and mixed numbers to improper fractions.

II. Invert the terms of the Divisor, and proceed as in multiplication.

NOTES.—1. The dividend and divisor may be reduced to a common denominator, and the numerator of the dividend be divided by the numerator of the divisor; this will give the same result as the rule.

2. Use cancellation where practicable.

Exercises for the Slate.

1. Divide $\frac{5}{6}$ by $\frac{8}{9}$. Ans. $\frac{15}{16}$
2. Divide $\frac{5}{9}$ by $\frac{1}{6}$. $3\frac{1}{3}$
3. Divide $\frac{1}{3}$ by $\frac{7}{12}$. $\frac{4}{7}$
4. Divide $\frac{42}{54}$ by $\frac{24}{35}$. $1\frac{29}{216}$
5. Divide $\frac{1}{2}$ of $\frac{3}{4}$ of 6 by $\frac{2}{3}$ of $\frac{3}{4}$ of 5. $\frac{9}{10}$
6. Divide $\frac{6}{7}$ of $\frac{5}{7}$ of $\frac{1}{3}$ by $\frac{1}{2}$ of $\frac{2}{3}$ of 6. $\frac{5}{49}$
7. How many times is $\frac{4}{5}$ contained in $\frac{5}{6}$? $1\frac{1}{24}$
8. How many times is $\frac{1}{2}$ of $\frac{3}{4}$ contained in $\frac{3}{7}$ of $2\frac{1}{2}$? Ans. $2\frac{6}{7}$
9. What is the quotient of $\frac{1}{9}$ of $\frac{5}{8}$ of 36 divided by $1\frac{8}{9}$ times $\frac{3}{5}$? Ans. $2\frac{7}{34}$
10. What is the value of $\dfrac{4\frac{1}{2}}{5\frac{3}{8}}$

OPERATION.

$$\frac{4\frac{1}{2}}{5\frac{3}{8}} = \frac{\frac{9}{2}}{\frac{43}{8}} = \frac{9}{2} \div \frac{43}{8} = \frac{9}{\cancel{2}} \times \frac{\overset{4}{\cancel{8}}}{43} = \frac{36}{43} \text{ Ans.}$$

This example is only another form for expressing division of fractions; it is sometimes called a *complex fraction*, and the process of performing the division is called *reducing a complex fraction to a simple one.*

11. Find the value of $\dfrac{4\frac{1}{2}}{2\frac{1}{4}}$ Ans. 2
12. Find the value of $\dfrac{11\frac{3}{7}}{\frac{4}{7}}$ 20
13. What is the value of $\dfrac{\frac{1}{2} \text{ of } \frac{3}{4}}{\frac{1}{6} \text{ of } \frac{5}{7}}$ $3\frac{3}{20}$
14. What is the value of $\dfrac{\frac{2}{5} \text{ of } \frac{5}{6}}{\frac{2}{9} \text{ of } 4\frac{1}{2}}$ $\frac{1}{3}$
15. Divide $\dfrac{\frac{1}{2}}{\frac{3}{4}}$ by $\dfrac{2\frac{1}{3}}{2\frac{1}{4}}$ $\frac{9}{14}$
16. At $18\frac{3}{4}$ cents a dozen, how many dozen of eggs can you buy for $87\frac{1}{2}$ cents? Ans. $4\frac{2}{3}$ doz.

17. A grocer sold $15\frac{1}{2}$ pounds of soda for $93\frac{3}{4}$ cents; how much was that per pound? Ans. $6\frac{3}{62}$ cts.

18. If $\frac{2}{3}$ of a yard cost $\frac{5}{6}$ of a dollar, what will 1 yard cost? Ans. $\$1\frac{1}{4}$

19. How many times will $11\frac{1}{3}$ gallons of oil fill a can which holds $\frac{1}{8}$ of $\frac{5}{6}$ of 2 gallons? Ans. $54\frac{2}{5}$

REDUCTION OF DENOMINATE FRACTIONS.

146. A Denominate Fraction is a fraction whose integral unit is *one* of a denomination of some compound number. Thus, $\frac{3}{7}$ of an hour is a denominate fraction, the integral unit being one hour; so are $\frac{3}{5}$ of a mile, $\frac{2}{3}$ of a bushel, &c., denominate fractions.

CASE I.

147. *To reduce a fraction of a higher denomination to an equivalent fraction of a lower denomination.*

EXAMPLE.—Reduce £$\frac{2}{720}$ to the fraction of a penny.

OPERATION.

$$\overset{£}{\frac{2}{720}} \times \frac{20}{1} \times \frac{12}{1} = \frac{480}{720} = \frac{2}{3}\text{d. Ans.}$$

OR,

$$\begin{array}{r|l} 8 \quad \cancel{60} & \\ \cancel{720} & 2 \\ & \cancel{20} \\ & \cancel{12} \\ \hline 3 & 2 = \frac{2}{3}, \text{Ans.} \end{array}$$

ANALYSIS.—To reduce pounds to pence, we must multiply by 20, and 12, the numbers in the table of money. And since the given number is a fraction of a pound, we indicate the process as in multiplication of fractions, and after cancelling, obtain $\frac{2}{3}$, the answer. Hence the following—

RULE. Multiply the fraction of the higher denomination by the numbers in the table, successively, between the given and required denominations.

Exercises for the Slate.

1. Reduce $\frac{4}{217}$ of 1 lb. avoirdupois to the fraction of an ounce. Ans. $\frac{64}{217}$ oz.

2. Reduce $\frac{25}{87}$ of a day to the fraction of an hour. Ans. $6\frac{78}{87}$ hours.

3. Reduce $\frac{6}{2784}$ of 1 mile to the fraction of a pole. Ans. $\frac{20}{29}$ pole.

4. Reduce $\frac{1}{80}$ of 1 bushel to the fraction of a pint.
Ans. $\frac{4}{5}$ pt.

5. Reduce $\frac{1}{3}$ of $\frac{2}{9}$ of 1 pound, avoirdupois, to the fraction of an ounce. Ans. $\frac{32}{27}$ or $1\frac{5}{27}$ oz.

6. Reduce $\frac{8}{9}$ of $\frac{1}{8}$ of 2 pounds to the fraction of an ounce Troy. Ans. $\frac{8}{3}$ oz.

CASE II.

148. *To reduce a fraction of a lower denomination to an equivalent of a higher denomination.*

EXAMPLE.—Reduce $\frac{2}{3}$ of a penny to the fraction of £1.

OPERATION.

$\frac{2}{3}\times\frac{1}{12}\times\frac{1}{20}=\frac{2}{720}=\frac{1}{360}$£, Ans.

OR,

$$\begin{array}{r|l} 3 & \cancel{2} \\ 6\ \cancel{12} & \\ 20 & \\ \hline 360 & 1 \end{array} = \frac{1}{360}\text{£ Ans.}$$

ANALYSIS.—To reduce pence to pounds, we must divide by 12 and 20, the numbers in the table. And since the given number of pence is a fraction, we indicate the process, as in division of fractions, and cancelling, obtain $\frac{1}{360}$, the answer. Hence the following—

RULE. Divide the fraction of the lower denomination by the numbers in the table, successively, between the given and required denominations.

Exercises for the Slate.

1. Reduce $\frac{1}{6}$ of a foot to the fraction of a yard.
Ans. $\frac{1}{18}$ yd.

2. Reduce $\frac{3}{8}$ of a yard to the fraction of a mile.
Ans. $\frac{3}{14080}$ mile.

3. Reduce $\frac{3}{4}$ of a pound to the fraction of 1 cwt. (112 lbs.)
Ans. $\frac{3}{448}$ lb.

4. What part of a pound is $\frac{3}{8}$ of a dram?
Ans. $\frac{3}{1280}$ lb.

5. What part of a bushel is $\frac{4}{5}$ of a pint? Ans. $\frac{1}{80}$ bus.

6. What fraction of a day is $6\frac{78}{87}$ hours? Ans. $\frac{25}{87}$ days.

CASE III.

149. *To find the value of a fraction in whole numbers of a lower denomination.*

EXAMPLE.—Find the value of $\frac{17}{29}$ of a cwt. (long weight).

OPERATION.

$$\begin{array}{rlll} & \text{cwt.} & \text{cwt. qrs.} & \text{lbs.} \\ 29) & 17\ (& 0\quad 2 & 9\frac{19}{29} \\ & \underline{\quad 4} & & \\ & 68 & & \\ & \underline{58} & & \\ & 10 & & \\ & \underline{\ 28} & & \\ & 280 & & \\ & \underline{261} & & \\ & \frac{19}{29} & & \end{array}$$

ANALYSIS.—Since $\frac{17}{29}$ cwt. is the same as $\frac{1}{29}$ of 17 cwt., we divide 17 cwt. by 29 as in division of compound numbers, (**112**,) and obtain for the answer 2 qrs. $9\frac{19}{29}$ lbs. Hence the following—

RULE. Consider the numerator of the given fraction as so many units of the given denomination, and divide by the denominator.

Exercises for the Slate.

Find the value of the following fractions.

1.	$\frac{3}{8}$ of a week.	Ans. 2 da. 15 h.
2.	$\frac{5}{6}$ of a month.	3 wk. 2 da. 8 h.
3.	$\frac{6}{7}$ of $\frac{3}{4}$ of 4 cwt. (long wt.)	2 cwt. 2 qrs. 8 lbs.
4.	$\frac{3}{4}$ of $\frac{1}{2}$ of 6 cwt.	2 cwt. 1 qr.
5.	$\frac{5}{6}$ of an acre.	3 ro. $13\frac{1}{3}$ po.
6.	$\frac{1}{2}$ of $\frac{3}{5}$ of £2.	£0 12s.
7.	$\frac{3}{8}$ of $3\frac{2}{3}$ acres.	1 ac. 1 ro. 20 po.
8.	$\frac{2}{11}$ of $1\frac{1}{7}$ of a pound, Apoth.	2 oz. 3 drs. 2 scr. $16\frac{68}{77}$ grs.
9.	$\frac{18}{26}$ of a day.	16 h. 36 min. $55\frac{5}{13}$ sec.

CASE IV.

150. *To reduce a compound number to a fraction of a higher denomination.*

EXAMPLE.—What part of £2 is 6 shillings and 3 pence?

OPERATION.

6s. 3d. = 75 pence.
£2 = 480 pence.
$\frac{75}{480} = \frac{5}{32}$ Ans.

ANALYSIS.—To find what part one compound number is of another, they must be reduced to the same denomination. In 6s. 3d there are 75 pence, and in £2 there 480 pence. Since 1 penny is $\frac{1}{480}$ of £1, 75 pence is $\frac{75}{480} = \frac{5}{32}$ of £2. Hence the following rule:

RULE. I. Reduce both quantities to the lowest denomination contained in either.

II. Then place that quantity which is to be the fraction of the other as numerator, and the remaining quantity as denominator.

Exercises for the Slate.

1. Reduce $4\frac{2}{3}$ shillings to the fraction of a pound. Ans. £$\frac{7}{30}$
2. Reduce 4s. 7d. to the fraction of £1. £$\frac{11}{48}$
3. Reduce 9s. $7\frac{1}{2}$d. to the fraction of £7 12s. 6d. £$\frac{77}{1220}$
4. What part of 1 lb. Troy is 16 dwt. 3 grs.? $\frac{48}{640}$ lb. Troy.
5. What part of 1 yd. is 2 ft. 4 in.? $\frac{7}{9}$ yd.
6. What part of 2 po. 4 yd. is $1\frac{1}{2}$ feet? $\frac{1}{30}$
7. Reduce $\frac{4}{5}$ of 1 pt. to the fraction of 1 gal. $\frac{1}{5}$ gal.
8. Reduce $\frac{7}{8}$ of 1 hour to the fraction of a day. $\frac{7}{192}$ day
9. What part of 10 bu. is 10 qts.? $\frac{1}{32}$
10. From a piece of land containing 4 ac. 2 ro. a farmer took 1 ro. 15 po. for a garden; what part of the whole did he take? Ans. $\frac{11}{144}$

REDUCTION OF DECIMALS.

CASE I.

151. *To reduce a decimal to a common fraction.*

EXAMPLE.—Reduce .125 to its equivalent common fraction.

OPERATION.

$.125 = \frac{125}{1000} = \frac{1}{8}$.

ANALYSIS.—We omit the decimal point, supply the proper denominator to the decimal, and then reduce the common fraction thus formed to its lowest terms. Hence the following—

RULE. Omit the decimal point, and supply the proper denominator.

Exercises for the Slate.

Reduce the following to common fractions—

1.	.1674	Ans. $\frac{837}{5000}$	7.	.625	Ans. $\frac{5}{8}$
2.	.125	$\frac{1}{8}$	8.	.00375	$\frac{3}{800}$
3.	.468	$\frac{117}{250}$	9.	.875	$\frac{7}{8}$
4.	.008	$\frac{1}{125}$	10.	.0095	$\frac{19}{2000}$
5.	.725	$\frac{29}{40}$	11.	.1876	$\frac{469}{2500}$
6.	.9375	$\frac{15}{16}$	12.	.1005	$\frac{201}{2000}$

CASE II.

152. *To reduce a common fraction to a decimal.*

EXAMPLE 1.—Reduce $\frac{5}{8}$ to its equivalent decimal.

FIRST OPERATION.

$\frac{5}{8} = \frac{5000}{8000} = \frac{625}{1000}$ .625, Ans.

SECOND OPERATION.

$$8)\underline{5.000}$$
$$.625$$

ANALYSIS.—We first annex the same number of ciphers to both terms of the fraction, this does not alter its value. We then divide both resulting terms by 8, the significant figure of the denominator, to obtain the *decimal* denominator, 1000. Then the fraction is changed to the decimal form by omitting the denominator. If the intermediate steps be omitted, the true result may be obtained as in the second operation.

EXAMPLE 2.—Reduce $\frac{3}{32}$ to its equivalent decimal.

OPERATION.

$$32)\underline{3.00000}$$
$$.09375, \text{ Ans.}$$

ANALYSIS.—Dividing as in the former example, we obtain a quotient of 4 figures, 9375. But since we annexed 5 ciphers, there must be 5 places in the required decimal; hence we prefix one cipher. From these illustrations we derive the following

RULE. I. Annex ciphers to the numerator and divide by the denominator.

II. Point off as many decimal places in the result as are equal to the number of ciphers annexed.

NOTE.—A common fraction can be reduced to an *exact* decimal when its lowest denominator contains only the prime factors 2 and 5, and not otherwise.

Exercises for the Slate.

Reduce the following fractional quantities to decimals—

1.	$\frac{1}{2}$	Ans. .5	6.	$\frac{17}{256}$	Ans. .06640625
2.	$\frac{3}{4}$	.75	7.	$\frac{19}{128}$	.1484375
3.	$\frac{7}{8}$	.875	8.	$\frac{13}{64}$	.203125
4.	$\frac{3}{16}$	.1875	9.	$\frac{5}{512}$	.009765625
5.	$\frac{15}{40}$	.375	10.	$\frac{3}{128}$	.0234375

11. Reduce $\frac{1}{6}$ to a decimal. Ans. 0.1666 +

12. Reduce $\frac{41}{333}$ to a decimal. 0.123123 +

NOTE. 1. The answers to the last two examples are called *repeating decimals*. The figure 6 in the 11th example, and the figures 123 in the 12th, are called *repetends*, because they are repeated, or occur in regular order. The sign + indicates that there is still a remainder.

2. A repetend has a point placed over the first and last figures to mark where it begins and ends.

CASE III.

153. *To reduce a denominate decimal to whole numbers of lower denominations.*

EXAMPLE.—Reduce £.675 to shillings and pence.

OPERATION.

.675
20
13,500
12
6,000

Ans. £0 13s. 6d.

ANALYSIS.— We first multiply by 20 to reduce the given number from pounds to shillings, and the result is 13 shillings and the decimal .500 of a shilling. We then multiply this decimal by 12 to reduce it to pence, and get 6 pence. Hence the answer is 13s. 6d.

RULE. I. Multiply the given decimal by that number in the table which will reduce it to the next lower denomination, and point off as in multiplication of decimals.

II. Proceed with the decimal part of the product in the same manner, until reduced to the required denominations. The integers on the left of the decimal point will be the answer required.

Exercises for the Slate.

Find the value of the following decimals.

1.	£.725.	Ans. £0 14s. 6d.
2.	.125 cwt. (short weight).	12 lb. 8 oz.
3.	.435 lbs. (avoir.)	6 oz. $15\frac{9}{25}$ drs.
4.	.4826 gal.	1 qt. 1 pt. 3.4432 gi.
5.	.845 hours.	50 min. 42 sec.
6.	.67 of a league.	2 m. 3 po. 1 yd. $3\frac{3}{5}$ in.
7.	.78875 of a long ton.	15 cwt. 3 qrs. 2 lb. 12.8 oz.
8.	.965625 of a mile.	7 fur. 29 po.
9.	.815625 of a pound Troy.	9 oz. 15 dwt. 18 grs.
10.	.07 of £2 10s.	3s. 6d.
11.	.0474609375 of £10 13s. 4d.	10s. $1\frac{1}{2}$d.
12.	.875 of £3 5s. 6d.	£2 17s. $3\frac{3}{4}$d.

CASE IV.

154. *To reduce a compound number to a decimal of a higher denomination.*

EXAMPLE.—Reduce 3 qts. 1 pt. 3 gills to the decimal of a gallon.

OPERATION.

4	3.00
2	1.750
4	3.87500
	.96875 gal. Ans.

OR,

3 qts. 1 pt. 3 gills = 31 gills.
1 gal. = 32 gills.
$\frac{31}{32}$ = .96875 gal. Ans.

ANALYSIS.—Since 4 gills make 1 pint, 2 pints make 1 quart, and 4 quarts 1 gallon, there will be $\frac{1}{4}$ as many pints as gills, $\frac{1}{2}$ as many quarts as pints, and $\frac{1}{4}$ as many gallons as quarts.—Or we may reduce 3 qts. 1 pt. 3 gills to the fraction of a gallon (as in **150**), and we have $\frac{31}{32}$ of a gallon, which reduced to a decimal equals .96875. Hence the following—

RULE. I. Divide the lowest denominaton given by that number in the table which will reduce it to the next higher, and annex the quotient as a decimal to that higher.

II. Proceed in the same manner until the whole is reduced to the denominaton required. Or,

Reduce the given number to a fraction of the required denominaton (150), and reduce this fraction to a decimal.

Exercises for the Slate.

Reduce

1. £0 7s. $4\frac{4}{5}$d. to the decimal of £1. Ans. .£.37
2. 10s. $0\frac{3}{4}$d. to the decimal of £1. £.503125
3. 3 pks. 1.12 qt. to the decimal of a bushel. .785 bu.
4. 10 oz. 13 dwt. 9 grs. to the decimal of 1 lb. Troy. Ans. .8890625 lb.
5. 2 oz. 13 dwt. to the decimal of 1 lb. $.2208\dot{3}$ lb.
6. 4 lb. 2 sc. to the decimal of 1 oz. $48.08\dot{3}$ oz.
7. 4 da. 18 hrs. to the decimal of 1 week. $.6\dot{7}85714\dot{2}$ wk.
8. $2\frac{1}{8}$ inches to the decimal of $2\frac{1}{5}$ miles. .000015 +
9. $3\frac{1}{2}$ acres to the decimal of $3\frac{1}{4}$ sq. yards. $5212.\dot{3}0769\dot{2}$
10. $\frac{4}{5}$ of a crown to the decimal of 21s. $.14\dot{2}8\dot{5}$

PROMISCUOUS EXERCISES IN THE PRECEDING RULES.

1. Reduce $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and 6 to fractions having a common denominator. Ans. $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$, $\frac{360}{60}$

2. What is the value of .75 of an ell English? Ans. 3 qr. 3 nails.

3. Add $4\frac{1}{2}$, $3\frac{1}{3}$, $5\frac{1}{5}$, $\frac{3}{8}$ of $8\frac{1}{2}$, and $\frac{1}{12}$. Ans. $15\frac{21}{80}$

4. What number multiplied by $\frac{3}{8}$ will produce $1141\frac{11}{64}$? Ans. $3043\frac{1}{8}$

5. If the dividend be $\frac{3}{4}$ and the quotient $\frac{1}{8}$, what is the divisor? Ans. 6

6. If $\frac{3}{10}$ of a barrel of flour cost \$2.34, what will be the cost of a whole barrel. Ans. \$7.80

7. If the smaller of two fractions be $\frac{24}{31}$, and their difference $\frac{7}{93}$, what is the greater? Ans. $\frac{79}{93}$

8. Find the difference between $\frac{2}{3}$ of $6\frac{7}{10}$ and $\frac{5}{9}$ of $4\frac{8}{15}$. Ans. $1\frac{128}{135}$

9. Reduce $\frac{4}{\frac{1}{6}}$ and $\frac{2\frac{1}{3}}{1\frac{1}{4}}$ to their simplest form. Ans. 24 and $1\frac{13}{15}$

10. Find the difference between $\frac{3}{4}$ of $5\frac{1}{5}$ and $\frac{1}{8}$ of $2\frac{3}{4}$. Ans. $3\frac{89}{160}$

11. Reduce $\frac{2}{3}$ of 13s. 6d. to the decimal of £1. Ans. £.45

12. Reduce 7 guineas to the decimal of £5 10s. 11d. Ans. 1.3251 +

13. From the sum of $\frac{1}{4}$, $\frac{1}{6}$, $\frac{2}{8}$, and $3\frac{1}{4}$ take the sum of $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$, and $\frac{1}{6}$ of $\frac{5}{8}$ and multiply the difference by $\frac{1}{5}$ of $3\frac{1}{2}$. Ans. $2\frac{3157}{7200}$

14. Change $\frac{5}{7}$ to an equivalent fraction having 91 for its denominator. Ans. $\frac{65}{91}$

15. At $\frac{1}{8}$ of $3\frac{1}{2}$ dollars per bushel, how many bushels of apples can be bought for $\$6\frac{1}{2}$? Ans. $14\frac{6}{7}$ bu.

16. A man owning $\frac{2}{3}$ of a factory sold $\frac{1}{3}$ of his share for $\$901\frac{1}{4}$; what was the whole value of the factory? Ans. $\$4055\frac{5}{8}$

17. What number diminished by the difference between $\frac{3}{4}$ and $\frac{3}{5}$ of itself, leaves a remainder of 34? Ans. 40

18. Find the sum of $\frac{2\frac{1}{2}}{5}$ of $7\frac{3}{4}$ and $1\frac{3}{4} \div 2\frac{1}{3}$. Ans. $4\frac{95}{136}$

19. Simplify $\left\{\frac{3}{4}+\frac{7}{6}\text{ of }5\frac{1}{2}\right\}\times\left\{\frac{5}{6}+\frac{2}{3}+3\frac{3}{4}\right\}$ Ans. $37\frac{5}{8}$

20. Simplify $\frac{4}{5}$ of $\frac{1}{2}$ — $\frac{2}{3}$ of $\frac{9}{17}$ + $\frac{3}{5}$ of $1\frac{10}{17}$. Ans. 1

21. If $\$7\frac{1}{4}$ will buy $3\frac{1}{4}$ cords of wood, how many cords can be bought for $\$10\frac{1}{2}$? Ans. $4\frac{41}{58}$

22. What is the sum of $\frac{1}{7}$ of a yard, $\frac{1}{7}$ of a foot, and $\frac{1}{7}$ of an inch? Ans. 7 inches.

23. If 3 tons of hay cost $49, what will $7\frac{4}{11}$ tons cost? Ans. $\$120.27\frac{3}{11}$

24. A man sold .15 of an estate to one person and then $\frac{5}{17}$ of the remainder to another person; what part of the estate did he still retain? Ans. $\frac{3}{5}$

25. Express $\frac{1}{2}$ $(6\frac{1}{2}+2\frac{2}{3}-3)$ as a decimal. Ans. $3.08\dot{3}$

26. Add together $\frac{3}{5}$ of a day, $\frac{2}{3}$ of an hour, and $\frac{4}{5}$ of 6 hours; and express the result as the decimal of a week. Ans. $.11\dot{8}2539\dot{6}$

27. A man sold 1 ton of hay for $12, and received $\frac{1}{2}$ the amount in sugar, at $\$\frac{1}{8}$ a pound, $\frac{1}{3}$ in money, and the remainder in molasses at $\$\frac{2}{5}$ a gallon; how many pounds of sugar, and how many gallons of molasses did he receive?

Ans. 48 lb. sugar.
5 gal. molasses.

28. A man gave $\frac{2}{3}$ of $1\frac{1}{5}$ times his ready money for a buggy, $\frac{3}{4}$ of what was left for a set of harness, and had $12 remaining; what did he pay for the buggy? Ans. $192

29. Express $\frac{3}{8}$ of a crown + $\frac{4}{5}$ of a shilling as a decimal of 7 shillings. Ans. $.38\dot{1}4285\dot{7}$

30. Reduce $\frac{21}{15000}$ of a year to the decimal of a day. Ans. .511

PRACTICE.

Example.—Find the price of 286 yards of cloth at £1 5s. $7\frac{1}{2}$d. per yard.

If we first find the price at £1, then at 5s., and at $7\frac{1}{2}$d., and add these three results, we shall have the price at £1 5s. $7\frac{1}{2}$d.

Now the price of 286 yards at £1 being £286, the price at 5s. will be $\frac{1}{4}$ of that, or £71 10s.; and the price at $7\frac{1}{2}$d. will

be $\frac{1}{8}$ of the price at 5s., that is £8 18s. 9d. Adding these three results, we find the price at £1 5s. $7\frac{1}{2}$d. to be £366 8s. 9d.

The operation may be written thus:—

	£	s.	d.		£	s.	d.
Price of 286 yards at	£1	0	0		£286	0	0
Price " " "	0	5	0	$\frac{1}{4}$	71	10	0
Price " " "	0	0	$7\frac{1}{2}$	$\frac{1}{8}$	8	18	0
Price of 286 yards at	£1	5	$7\frac{1}{2}$		£366	8	9

The answer to this question might be found by compound multiplication: but the process is longer. The method of finding prices by aliquot parts is therefore commonly practised; hence it is called "Practice."

155. From the preceding operation we perceive that **Practice** is a short, or compendious, method of finding the value of any quantity, or number of articles, when the price of a unit of any denomination is given.

156. An Aliquot part of a quantity is such a part as, when taken a certain number of times will *exactly* make that quantity.

Preliminary Exercises.

1. Make a table of aliquot parts of a penny, a shilling, and a pound.

2. In the following list of aliquot parts name what part each is of another denomination. Thus—What is 3s. 4d.? One sixth of a £.

3s. 4d., 2s. 6d., 10s., 2s., 3d., 4d., 6d., $1\frac{1}{2}$d., $\frac{1}{4}$d., 2s. 6d., 5s., $7\frac{1}{2}$d., 2 cwt., 15 cwt., 7 lbs., 2 qrs., 2 gals., 4 pks.

3. What part of

2s. is 8d.	10s. is 1s. 3d.	11s. is 11d.	£2 is 5s.
2s. 4d. " 4d.	5s. " 5d.	1s. " $1\frac{1}{2}$d.	£2 " 8s.
6s. 8d. " 1s. 8d.	5s. 6d " $5\frac{1}{2}$d.	9s. 6d. " $9\frac{1}{2}$d.	£2 " 10s.
10s. " 2s. 6d.	8s. 6d " $8\frac{1}{2}$d.	3d. " $\frac{1}{4}$	£4 " 16s.
7s. " 7d.	1s. 3d " 5d.	9d. " $4\frac{1}{2}$d.	£4 " 6s. 8d.
8s. " 8d.	4s. " 2s. 4d.	12s. 6d. " 2s. 6d.	£8 " 7s. 6d.
10s. " 3s. 4d.	13s. 6d " $3\frac{1}{2}$d.	£1 " 2s. 6d.	£2 " 12s. 6d.
4s. " 8d.	3s. 4d " 8d.	10s. " 1s. 3d.	8s. 9d. " $8\frac{3}{4}$d.

4. What is the

$\frac{1}{2}$ of 8s.	$\frac{1}{8}$ of 16s.	$\frac{3}{4}$ of £1	$\frac{1}{8}$ of £4
$\frac{1}{5}$ " 10s.	$\frac{1}{12}$ " 4s.	$\frac{3}{4}$ " 5s.	$\frac{1}{8}$ " £5
$\frac{1}{8}$ " 3s. 4d.	$\frac{1}{12}$ " 7s. 9d.	$\frac{1}{7}$ " 7s.	$\frac{1}{4}$ " £5
$\frac{1}{4}$ " 10s.	$\frac{3}{4}$ " 6d.	$\frac{2}{3}$ " £1	$\frac{1}{6}$ " £6

5. Give the aliquot parts for

12s.	6s. 3d.	3s. $7\frac{1}{2}$d.	1 ro. 4 po.
14s.	12s. 6d.	15s. $7\frac{1}{2}$d.	2 ro. 15 po.
13s. 9d.	17s. 6d.	17s. 9d.	3 ro. 59 pd.
15s.	18s. 2d.	16s. 3d.	3 ro, $37\frac{1}{2}$ po,
6s.	8s. 4d.	10s. 10d,	5 dwt. 9 grs.
3s. 9d.	7s. 4d.	0s. $5\frac{1}{2}$d.	3 drs. 5 grs.
3s.	12s. 8d.	0s. $10\frac{3}{4}$d.	3 qrs. 27 lbs.
14s.	14s. 8d.	16s. $3\frac{1}{2}$d.	2 qrs. 17 lbs.
12s. 2d.	5s. $2\frac{1}{2}$d.	1s. $1\frac{1}{2}$d.	1 qr. $26\frac{1}{2}$ lbs.

CASE I,

157. *To find the value, when the given quantity is a simple number, and the price less than 1 shilling.*

EXAMPLE 1.—Calculate the price of 44 articles at 7d.

OPERATION.

44 at 1d. = 3s. 8d.
44 at 7d, = 7 times 3s. 8d. = £1 5s. 8d.

OR,

	44 at 7d.		
6d. is $\frac{1}{2}$ of 1s.	22	0	= price at 6d,
1d. is $\frac{1}{6}$ of 6d.	3	8	= price at 1d.
	£1 5	8	= price at 7d.

From the above illustration we have the following—

RULE.—Find the price at 1 penny, and multiply by the pence in the price. Or,
Find the price by means of aliquot parts.

Exercises for the Slate.

Calculate the value of the following articles.

1. 24 at 3d. and at 9d.	7. 126 at 10d. and at 2d.
2. 36 " 7d, and " 5d.	8. 133 " 11d. and " 1d.
3. 46 " 8d. and " 4d.	9. 237 " 9d. and " 3d.
4. 63 " 10d. and " 2d.	10. 187 " 8d. and " 4d.
5. 72 " 11d. and " 1d.	11. 483 " 7d. and " 5d.
6. 65 " 5d. and " 7d.	12. 209 " 5d. and " 7d.

Example 2.—Find the price of 126 at $7\frac{1}{2}$d. each.

OPERATION.

126 at 1d. = 10s. 6d.

126 at $7\frac{1}{2}$ = $7\frac{1}{2}$ times 10s. 6d. = £0 10 6

	£0	10	6
			$7\frac{1}{2}$
	0	5	3
	6	13	3
	£3	18	9

OR,

126 at $7\frac{1}{2}$d.

6d. is $\frac{1}{2}$ of 1s.	63	0	= price at 6d.
$1\frac{1}{2}$d, is $\frac{1}{4}$ of 6d.	15	9	= price at $1\frac{1}{2}$d.
	£3 18	9	= price at $7\frac{1}{2}$d.

13. 48 at $7\frac{1}{2}$d, and at $4\frac{1}{2}$d.
14. 89 " $9\frac{1}{2}$d. and " $2\frac{1}{2}$d.
15. 72 " $7\frac{3}{4}$d. and " $4\frac{1}{4}$d.
16. 126 " $1\frac{1}{4}$d. and " $10\frac{3}{4}$d.
17. 173 " $5\frac{3}{4}$d. and " $6\frac{1}{4}$d.
18. 365 " $8\frac{1}{2}$d. and " $3\frac{1}{2}$d.
19. 246 at $1\frac{3}{4}$d. and at $10\frac{1}{4}$d.
20. 239 " $3\frac{1}{2}$d. and " $8\frac{1}{2}$d.
21. 101 " $5\frac{1}{4}$d. and " $6\frac{3}{4}$d.
22. 196 " $7\frac{3}{4}$d. and " $4\frac{1}{4}$d.
23. 365 " $9\frac{1}{8}$d. and " $2\frac{7}{8}$d.
24. 494 " $6\frac{5}{8}$d. and " $5\frac{3}{8}$d.

Note.—All the exercises given under this and subsequent rules should be worked by dollars and cents also, and thus verify the results.

CASE II.

158. *To find the value when the given quantity is a simple number, and the price given in shillings.*

Example 1.—Find the price of 322 yds. at 6s. per yard.

OPERATION.

322 at 1s. = £16 2s.

322 at 6s. = 6 times £16 2 = £96 12s.

OR,

Multiplying by half the price and doubling the unit figure for shillings; thus,

322 at 6s.

3

£96 12 Ans. as before

EXAMPLE 2.—Find the price of 137 yards at 17 shillings per yard.

OPERATION.

$$\begin{array}{rl}
137 & \\
8\frac{1}{2} & = \frac{17}{2} \\
\hline
68 & \text{rem.} = 1. \\
1096 & \text{twice } 4 = 8. \\
\hline
\pounds 116 \quad 9 \quad 0 & \text{Answer.}
\end{array}$$

From the above we derive the following

RULE.—Multiply by half the number of shillings; double the units figure of the product for shillings and take the others as pounds.

Exercises for the Slate.

Find the value of

1.	126 at 16s. and at 4s.	6.	384 at 4s. and at 16s.
2.	132 " 15s. and " 5s.	7.	596 " 9s. and " 11s.
3.	689 " 14s. and " 6s.	8.	1832 " 11s. and " 9s.
4.	128 " 18s. and " 2s.	9.	1596 " 12s. and " 8s.
5.	136 " 17s. and " 3s.	10.	1118 " 13s. and " 7s.

11. 1896 at 16s.	Ans. £1516 16.	14. 48 at 9s.	Ans. £21 12.
12. 1346 " 17s.	£1144 2.	15. 186 " 7s.	£65 2.
13. 1284 " 3s.	£192 12.	16. 327 " 11s.	£179 17.

CASE III.

159. *To find the value when the price consists of pounds and shillings.*

EXAMPLE.—What is the cost of 187 tons at £6 11s. per ton.

OPERATION.

$$\begin{array}{rll}
187 & & \\
65\frac{1}{2} & = \text{half the number of shillings in the price.} & \\
\hline
9,3 & \text{remainder} = & 1 \\
93,5 & \text{twice } 8 = & 16 \\
1122 & & \\
\hline
\pounds 1224 \quad 17 \quad 0 & & 17
\end{array}$$

Hence the

RULE. To the number of pounds annex half the number of shillings for a multiplier. Double the units figure of the product for shillings.

Exercises for the Slate.

Find the value of

(1) 426 at £7 8s. and at £2 12s.	(6) 563 at £6 7s. and at £3 13s.
(2) 446 " £4 3s. and " £5 17s.	(7) 851 " £8 13s. and " £1 7s.
(3) 642 " £5 7s. and " £4 13s.	(8) 754 " £6 17s. and " £3 3s.
(4) 741 " £6 9s. and " £3 11s.	(9) 694 " £4 15s. and " £5 5s.
(5) 684 " £9 13s. and " £0 7s.	(10) 339 " £5 15s. aad " £4 5s.

11.	183 at £2 13s.	Ans. £484 19s.
12.	129 " £7 15s.	£999 15s.
13.	486 " £8 18s.	$17301.60.
14.	596 " £9 19s.	$23720.80.

CASE IV.

160. *To find the value of any number of articles, when the price is given in shillings and pence, or in pounds, shillings and pence.*

EXAMPLE 1.—If 1 yard cost 16s. 3d., what will 127 yards cost at the same rate?

OPERATION.

	127 at 16s. 3d. per yard.							
10s. is $\frac{1}{2}$ of £1	63	10	0	= price at	£0	10	0	
5s. is $\frac{1}{2}$ of 10s.	31	15	0	= "	0	5	0	
1s. 3d. is $\frac{1}{4}$ of 5s.	7	18	9	= "	0	1	3	
	£103	3	9	= price at	£0	16	3	

EXAMPLE 2.—Find the price of 187 yards at £2 13s. 4d. per yard.

OPERATION.

	187 at £2 13s. 4d.							
	2							
	374	0	0	= price at	£2	0	0	per yard.
10s. is $\frac{1}{2}$ of £1	93	10	0	= "	0	10	0	"
3s. 4d. is $\frac{1}{3}$ of 10s.	31	3	4	= "	0	3	4	"
	£498	13	4	= price at	£2	13	4	

From the foregoing we have the following

RULE.—**Multiply the quantity by the pounds, if any, and ta' liquot parts for the shillings and pence.**

Exercises for the Slate.

(1) 132 at 3s. 9d. and at 16s. 3d.
(2) 156 " 3s 4d. and " 16s. 8d.
(3) 999 " 18s. 4d. and " 1s. 8d.
(4) 365 " 12s. 6d. and " 7s. 6d.
(5) 831 " 17s. 5d. and " 2s 7d.
(6) 144 " 11s. 7d. and " 8s. 5d.
(7) 127 at 5s. 7½d. and at 14s. 4½d.
(8) 395 " 12s. 2½d. and " 7s. 9½d.
(9) 987 " 12s. 1½d. and " 7s. 10½d.
(10) 1118 at 14s. 8¼d. and at 5s. 3¾d.
(11) 5639 " 18s. 4½d. and " 1s. 7½d.
(12) 3017 " 16s. 2¾d. and " 3s. 9¼d.

13.	2436 at 15s.	Ans. £1827 0s. 0d.
14.	2739 at 10s. 10d.	£1483 12s. 6d.
15.	4938 at 15s. 7½d.	£3857 16s. 3d.
16.	9852 at 15s. 11¼d.	£7850 16s. 3d.
17.	3482 at 19s. 11¼d.	£3471 2s. 4½d.
18.	9584 at 11s. 6¾d.	£5540 15s. 0d.
19.	7947 at 18s. 0¼d.	£7160 11s. 6¾d.
20.	543 at £1 8s. 8d.	£778 6s. 0d.
21.	296 at £2 13s. 4d.	£789 6s. 8d.
22.	568 at £2 18s. 4d.	£1656 13s. 4d.
23.	496 at £3 19s. 8¼d.	£1976 5s. 0d.
24.	524 at £8 18s. 11¾d.	£4689 5s. 1d.

CASE V.

161. *To find the price when the quantity contains a fraction.*

EXAMPLE.—What is the value of 136⅝ yards of cloth at 17s. 6d. per yard ?

OPERATION.

136⅝ at 17s. 6d.per yard.

	£	s.	d.	
	£136	12	6	= price at £ 1 per yard.
10s. = ½ £1	68	6	3	= price at 10s. "
5s. = ½ 10s,	34	3	1½	= price at 5s. "
2s. 6d. = ½ 5s.	17	1	6¾	= price at 2s. 6d. "
	£119	10	11¼	= price at 17s. 6d. "

SECOND OPERATION.

136⅝ at 17s. 6d.

	£	s.	d.	
10s. = ½ of £1	68	0	0	= price of 136 at 10s.
5s. = ½ of 10s.	34	0	0	= " " 5s.
2s. 6d. = ½ of 5s.	17	0	0	= " " 2s. 6d.
⅝ of 17s. 6d.	0	10	11¼	= " ⅝
	£119	10	11¼	= price of 136⅝ at 17s. 6d.

NOTE.—The price of ⅝ may also be found by means of aliquot parts.

THIRD OPERATION.

$136\frac{5}{8}$ at 17s. 6d.
8

1093

10s. $= \frac{1}{2}$ of £1	546	10	0
5s. $= \frac{1}{2}$ of 10s.	273	5	0
2s. 6d. $= \frac{1}{2}$ of 5s.	136	12	6
Dividing by 8)	956	7	6
	£119	10	$11\frac{1}{4}$

OR,

$136\frac{5}{8}$ at | 17s. 6d.
8 $\frac{1}{8}$ | 2 $2\frac{1}{4}$

1093

2s. $= \frac{1}{10}$ of £1	109	6	0
2d. $= \frac{1}{12}$ of 2s.	9	2	2
$\frac{1}{4}$d. $= \frac{1}{8}$ of 2d.	1	2	$9\frac{1}{4}$
	£119	10	$11\frac{1}{4}$

BY DOLLARS AND CENTS.

$146\frac{5}{8}$ at \$3.50

146.625
3.5

733125
439875

\$513.1875

or,

$146\frac{5}{8}$ at \$3.50
$1.46\frac{5}{8}$

21.00
140.0
350.
$\frac{4}{8} = \frac{1}{2}$ of 1 — 1.75
$\frac{1}{8} = \frac{1}{4}$ of $\frac{4}{8}$ — .4375

\$513.1875

Exercises for the Slate.

1. $284\frac{1}{2}$ at 10s. — Ans. £142 5s. 0d.
2. $354\frac{1}{2}$ " 13s. 4d. — £236 6s. 8d.
3. $968\frac{3}{4}$ " 15s. 6d. — \$2923.04
4. $279\frac{2}{7}$ " \$4.90. — \$1368.50.

5.	$512\frac{3}{8}$ at \$11.56¼.	\$5768.88
6.	$849\frac{2}{9}$ " \$15.71¼.	\$13343.40$\frac{5}{12}$.
7.	$440\frac{368}{417}$ at £2 12s. 1½d.	£1149 1s. 0d.
8.	$578\frac{3}{8}$ at \$19.56$\frac{3}{8}$.	\$11316.87$\frac{1}{12}$.
9.	$427\frac{7}{9}$ " £5 19s. 7½d.	£2558 12s. 11d.
10.	$651\frac{7}{10}$ " \$15.75.	\$10264.27½.
11.	$542\frac{5}{12}$ " £2 6s. 9d.	£1267 17s. 11¾d.
12.	$491\frac{4}{13}$ " £2 5s. 6d.	£1117 14s. 6d.

CASE VI.

162. *To find the value of a compound quantity when the price of a unit of the quantity is given in dollars and cents.*

EXAMPLE 1.—Find the value of 126 cwt. 3 qrs. 14 lbs. (long weight) at \$14.62½ per cwt.

OPERATION.

126 cwt. 3 qrs. 14 lbs. at \$14.625

	126		
	\$1842.75	=	price of 126 cwt
2 qrs. = ½ of 1 cwt.	7.3125	=	" 2 qrs.
1 qr. = ½ of 2 qrs.	3.65625	=	" 1 qr.
14 lbs. = ½ of 1 qr.	1.828125	=	" 14 lbs.
	\$.1855.546875	=	price of 126 cwt.,&c.

EXAMPLE 2.—What will 13 cwt. 2 qrs. 15 lbs. (short weight) of oatmeal cost, at \$3.75 per cwt.?

OPERATION.

13 cwt. 2 qrs. 15 lbs. at \$3.75 per 100 lbs.

	13		
	\$48.75	=	price of 13 cwt.
2 qrs. = ½ of 1 cwt.	1.875	=	" 2 qrs.
10 lbs. = ⅕ of 2 qrs.	.375	=	" 10 lbs.
5 lbs. = ½ of 10 lbs.	.1875	=	" 5 lbs.
	\$51.1875	=	price of 13 cwt., &c.

OR,

13 cwt. 2 qrs. 15 lbs. = 13.65 cwt. at \$3.75

$$
\begin{array}{r}
3.75 \\
\hline
6825 \\
9555 \\
4095 \\
\hline
\$51.1875
\end{array}
= \text{price as before.}
$$

NOTE.—In calculating, it will often be found more convenient to reduce the lower denominations to a decimal of a higher, and multiply as in decimals.

EXAMPLE 3.—Find the price of 14 ac. 3 ro. 35 po. at \$22.16$\frac{1}{5}$ per acre.

OPERATION.

14 ac. 3 ro. 35 po. at \$22.162 per acre.

$$
\begin{array}{r|ll}
 & 14 & \\
\hline
 & 310.268 & = \text{price of 14 ac.} \\
2 \text{ ro.} = \frac{1}{2} \text{ of } 1 \text{ ac.} & 11.081 & = \quad 2 \text{ ro.} \\
1 \text{ ro.} = \frac{1}{2} \text{ of } 2 \text{ ro.} & 5.5405 & = \quad 1 \text{ ro.} \\
20 \text{ po.} = \frac{1}{2} \text{ of } 1 \text{ ro.} & 2.77025 & = \quad 20 \text{ po.} \\
10 \text{ po.} = \frac{1}{2} \text{ of } 20 \text{ po.} & 1.385125 & = \quad 10 \text{ po.} \\
5 \text{ po.} = \frac{1}{2} \text{ of } 10 \text{ po.} & .6925625 & = \quad 5 \text{ po.} \\
\hline
 & \$331.7374375 & = \text{price of 14 ac., \&c.}
\end{array}
$$

OR,

14 ac. 3 ro. 35 po. = 14$\frac{31}{32}$ ac. = 14.96875 ac. at \$22.16$\frac{1}{5}$

$$
\begin{array}{r}
22.16\frac{1}{5} \\
\hline
299375 \\
8981250 \\
1496875 \\
2993750 \\
2993750 \\
\hline
\$331.7374375
\end{array}
\text{ Ans. as before.}
$$

From these illustrations we deduce the following general

RULE. **Multiply the price by the integral part of the quantity, then separate the remainder into aliquot parts of 1 of the quantity whose rate is given, or successively of each other, as the case may require.** Or,

Reduce the quantity to a decimal of the same denomination as the quantity whose rate is given, and multiply as in decimals.

Exercises for the Slate.

					Answers.
	cwt.	qrs.	lbs.	*(long weight.)*	
1.	163	3	14	at \$15.20.	\$2490.90
2.	115	2	17	at \$13.10$\frac{1}{2}$.	\$1515.6166+
3.	18	3	21	at \$14.18$\frac{1}{4}$.	\$268.581093
4.	136	2	27	at £2 19s. 6d.	£406 16s. 1$\frac{1}{8}$d.
5.	18	3	24$\frac{1}{2}$	at £5 15s. 5$\frac{1}{4}$d.	£109 9s. 8.46+
	(short weight.)				
6.	181	3	15	at £2 3s. 9d.	£397 18s. 1$\frac{1}{2}$d.
7.	165	2	22	at \$4.37$\frac{1}{2}$.	\$725.025
8.	172	3	18	at \$19.19.	\$3318.5267
9.	111	1	1	at \$4.33$\frac{1}{4}$.	482.03395
	ac.	ro.	po.		
10.	121	3	14	at \$15.61.	1901.883375
11.	136	2	19	at £2 14s. 5$\frac{1}{4}$d.	£371 17s. 2$\frac{127}{640}$
12.	183	1	38$\frac{1}{2}$	at \$15.55$\frac{1}{2}$.	\$2854.196+
	yds.	qrs.	nls.		
13.	15	3	1	at \$2.10.	\$33.206$\frac{1}{4}$
14.	16	2	3	at \$4.52$\frac{1}{2}$.	\$75.5109+
15.	28	3	3$\frac{1}{2}$	at \$14.10$\frac{1}{4}$.	\$408.5317+
	tons.	cwt.	qrs.		
16.	113	12	3	at \$14.62$\frac{1}{2}$.	\$1661.94$\frac{27}{32}$
17.	165	13	2	at \$22.80$\frac{1}{5}$.	\$3777.72$\frac{27}{200}$
18.	567	2	3	at \$12.33$\frac{1}{3}$.	\$6993.69$\frac{7}{12}$
19.	384	19	3$\frac{1}{2}$	at \$14.80.	\$5697.90$\frac{3}{4}$
20.	144	18	3$\frac{5}{8}$	at \$19.27$\frac{1}{2}$.	\$2793.82$\frac{23}{256}$

CASE VII.

163. *To find the value of a compound quantity when the price of a unit of the quantity is given in pounds, shillings and pence.*

EXAMPLE 1.—Find the price of 3 cwt. 2 qrs. 4 lbs., long weight, of flour, at £1 per cwt.

OPERATION.		
cwt.	qrs.	lbs.
3	2	4
	5	2$\frac{1}{7}$
£3	10s.	8$\frac{4}{7}$d.

ANALYSIS.—Since 1 cwt. costs £1, 3 cwt. will cost 3 times as much, or £3. Again 1 qr. will cost $\frac{1}{4}$ of £1, or 5s., and 2 qrs. will cost 2 times as much, or 10s. Lastly, 1 lb. will cost $\frac{1}{28}$ of 5s. or 2$\frac{1}{7}$d., and 4 lb. will cost 4 times 2$\frac{1}{7}$d., or 8$\frac{4}{7}$d.

EXAMPLE 2.—Find the value of 16 cwt. 3 qrs. 14 lbs., long weight, at £2 13s. 6d. per cwt.

OPERATION.

	cwt.	qrs.	lbs.				
	16	3	14				
		5	$2\frac{1}{7}$				
	£16	17	6	= price at £1 per cwt.			
			2				
	33	15	0	= price at	£2	0	0
10s. = $\frac{1}{2}$ of £1	8	8	9	= "	0	10	0
2s. 6d. = $\frac{1}{4}$ of 10s.	2	2	$2\frac{1}{4}$	= "	0	2	6
1s. = $\frac{1}{10}$ of 10s.	0	16	$10\frac{1}{2}$	= "	0	1	0
	£45	2	$9\frac{3}{4}$	= price at	£2	13	6

OR,

16 cwt. 3 qrs. 14 lbs. at £2 13 6

					cwt.	qrs.	lbs.
16 cwt. 3 qrs. 14 lbs. at	£2	13	6				
			16				
	42	16	0	= price of	16	0	0
2 qrs. = $\frac{1}{2}$ of 1 cwt.	1	6	9	= "	0	2	0
1 qr. = $\frac{1}{2}$ of 2 qrs.	0	13	$4\frac{1}{2}$	= "	0	1	0
14 lbs. = $\frac{1}{2}$ of 1 qr.	0	6	$8\frac{1}{4}$	= "	0	0	14
	£45	2	$9\frac{3}{4}$	= price of	16	5	14

From these examples we deduce the following general

RULE. Find the value of the quantity, if any, of which the rate is given, by Compound Multiplication, then separate the remainder of the quantity into aliquot parts, as in the former rule. Or,

Find the price of the given quantity at £1, by one of the following rules, then multiply the result by the pounds, if any, in the price and separate the shillings and pence into aliquot parts.

RULES.

In calculating the price of

1. *Hundreds, quarters and pounds, long weight, at* £1 *per cwt.*, multiply the pounds by $2\frac{1}{7}$ for pence, and the quarters by 5 for shillings.

2. *Hundreds, quarters and pounds, short weight, at* £1 *per cwt.*, multiply the pounds by $2\frac{2}{5}$ for pence, and the quarters by 5 for shillings.

3. *Tons, hundreds and quarters, at £1 per ton,* take the tons and hundreds as pounds and shillings, and multiply the quarters by 3 for pence.

4. *Acres, roods and poles, at £1 per acre,* multiply the poles by $1\frac{1}{2}$ for pence, and the roods by 5 for shillings.

5. *Yards, quarters and nails, at £1 per yard,* take each quarter at 5s. and each nail at 1s. 3d.

6. *Oz., dwts. and grains, Troy weight, at £1 per ounce,* take the ounces as pounds, the pennyweights as shillings, and half the grains as pence.

164. In calculating by means of aliquot parts, it will often be more convenient to use the decimal form of remainder instead of the common fractional. It will be sufficient to carry the decimals to two places, as in the following example.

EXAMPLE 3.—What will 126 ac. 3 ro. 15 po. cost at £2 11s. 3d. per acre?

OPERATION.

	126	3	15	at £2 11s. 3d.
		5	$1\frac{1}{2}$	
	£126	16	10.50	= price at £1 per acre.
			2	
	253	13	9.00	= price at £2 0 0 per acre
10s. = $\frac{1}{2}$ of £1	63	8	5.25	= " 0 10 0 "
1s. 3d. = $\frac{1}{8}$ of 10s.	7	18	6.66	= " 0 1 3 "
	£325	0	9(91	= price at £2 11 3 per acre)

NOTE.—In working by this method the penny is supposed to be divided into 100 equal parts. Hence .25d. = $\frac{1}{4}$, .50d. = $\frac{1}{2}$, .75d. = $\frac{3}{4}$

In valuing the decimal in the answer we consider to which of these it is nearest and value it accordingly.

General Exercises.

1.	18967 at $15.01.	Ans. $284694.67
2.	13468 at £2 18s.	$190078.38
3.	1768 at £9 13s.	£17061 4s. 0d.
4.	1476 at £11 15s.	$84402.60
5.	1367 at £3 19s. 6d.	£5433 16s. 6d.
6.	387 at $14.83½.	$5741.14½
7.	1429 at $18.62.	$26607.98
8.	148¼ at $11.10½.	$1646.31⅝

9.	$367\frac{1}{8}$			at £11 13s. 6d.		\$20859.44
10.	$463\frac{3}{5}$			at \$18.18$\frac{1}{2}$.		\$8430.56$\frac{3}{5}$
11.	$519\frac{11}{12}$			at £1 0s. 6d.		£532 18s. 3$\frac{1}{2}$d.
12.	$345\frac{13}{16}$			at \$6.72$\frac{1}{2}$.		\$2325.5890625
	cwt.	qrs.	lbs.	*(long weight,)*		
13.	15	3	16	at £0 13s. 6d.	per cwt.	£10 14s. 6.642+
14.	14	2	24$\frac{1}{2}$	at £3 18s. 6d.	"	£57 15s. 5.062d.
15.	19	3	23	at \$15.62$\frac{1}{2}$	"	\$311.80$\frac{1}{4}$ nearly.
				(short weight.)		
16.	17	3	15	at £3 15s. 6d.	"	£67 11s. 5.4d.
17.	19	3	14	at \$18.61$\frac{1}{4}$	"	\$370.20$\frac{1}{4}$
18.	23	3	11	at \$12.32$\frac{1}{2}$	"	\$294.07$\frac{1}{2}$ nearly.
19.	26	2	17$\frac{1}{2}$	at 19s. 7$\frac{1}{2}$d.	"	£26 3s. 5.96d.
20.	136	2	10$\frac{1}{4}$	at £3 16s.	"	£519 1s. 9.48d.
21.	48	1	27	at \$7.87$\frac{1}{2}$	"	\$382.095
	tons.	cwt.	qrs.			
22.	11	13	3	at £5 16s. 3d.	per ton.	£67 18s. 8.06d.
23.	14	17	2	at \$18.88	"	\$280.84
24.	13	14	1	at \$27.33	"	\$374.7626
25.	18	19	3$\frac{1}{2}$	at £2 19s. 7$\frac{1}{2}$d.	"	£56 12s. 6.05d.
26.	84	3	2$\frac{1}{4}$	at £11 3s. 4$\frac{1}{2}$d.	"	£940 3s. 3.4d.
	yd,	qrs.	nls.			
27.	15	3	1	at \$2,18	per yd.	\$34.47125
28.	18	2	3	at \$11.16	"	\$208.55$\frac{1}{4}$
29.	15	1	2	at 13s. 9$\frac{1}{2}$d.	"	£10 12s. 0.56d.
30.	25	3	2	at 18s. 11d.	"	£24 9s. 5.62d.
31.	27	3	1$\frac{1}{2}$	at \$4.16$\frac{1}{2}$	"	\$115.969
	ac.	ro.	po.			
32.	126	3	14	at £2 19s. 8d.	per acre.	£378 7s. 11$\frac{33}{40}$d.
33.	384	1	27	at \$18.55	"	\$7130.96+
34.	361	2	19	at \$18.27$\frac{1}{2}$	"	\$6608.58+
35.	84	1	37$\frac{1}{2}$	at \$10.19	"	\$860.895+
36.	172	1	15	at £1 18s. 9d.	"	£333 18s. 3$\frac{27}{32}$d.
	oz.	dwt.	grs.			
37.	14	12	9	at \$1.62	per oz.	\$23.682+
38.	17	3	19	at \$18.50	"	\$318.007+
39.	12	13	20	at £2 3s. 6$\frac{1}{2}$d.	"	£27 12s. 7.39d.
40.	15	11	16$\frac{1}{2}$	at £1 19s. 6d.	"	£30 15s. 6.993d.+
	gal.	qts.	pts.			
41.	13	3	1	at 13s. 6d.	per gal.	£9 7s. 3$\frac{3}{4}$d.
42.	18	3	0	at \$1.10	"	\$20.62$\frac{1}{2}$
43.	27	1	1	at \$14.16	"	\$387.63
44.	9	1	0	at 1s. 9d.	per quart.	\$15.75

PROPORTION.

165. In the foregoing exercises on the Rules of Practice there are apparently only two terms given, the price and quantity; but in each there are really three things given.

Taking the last exercise as an example, it may be written thus:—

If 1 quart of oil cost 1s. 9d., what is the cost of 37 quarts?

ANALYSIS.—Here the price of a certain quantity is given, and we wish to know the price of so many times that quantity. 37 quarts are 37 times 1 quart, therefore the price of 37 quarts will be 37 times the price of 1 quart; that is 1s. 9d. × 37 = £3 4s. 9d., or $15.75.

EXAMPLE 2.—If 6 lbs. of tea cost 18s. 9d., what is the cost of 48 lbs.?

ANALYSIS.—Here the price of 6 lbs is given, and we wish to know the price of 48 lbs. 48 lbs. are 8 times 6 lbs., therefore the price of 48 lbs. will be 8 times the price of 6 lbs.; that is 18. 9d. × 8 = £7 10s.

Questions of this sort, in which the quantity whose price is sought in so many times the quantity whose price is given, are generally solved by Multiplication. In all such questions there are three numbers given, two being of the same kind, and the third of a different kind; hence it is sometimes called the "Rule of Three."

A fourth quantity is in all cases sought, which is of the the same kind as the third given.

Exercises for the Slate.

1. If 5 yards cost £9, what will 20 yards cost?
2. If 3 yds. cost £2 5s. 5¼d., what will 24 yards cost?
3. How much must be paid for 32 yds., if 4 yards cost £6 16s. 4d.
4. If a man walk 81 miles in 3 days, how far will he walk in 15 days?
5. If 2 quarts cost $1.53, what cost 2 gallons?
6. The wages of 8 men amount to £7 6s. 5½d, what will the wages of 128 men amount to?
7. If ½ lb. of tea cost 22½ cents, what cost 8 lbs.?

8. How many yds. of cloth at 3s. 6d. are worth 27 yds. at 14s. per yard?

9. If 8 yards cost £2 8s., what is the price of 2 yards?

ANALYSIS.—Here the quantity whose price is sought is an even part of that whose price is given.

Since 2 yards is the fourth part of 8 yards, the price of 2 yards will be the fourth part of that of 8 yards.

Now $\frac{1}{4}$ of £2 8s. = £0 12s., which is the answer required.

Such questions are solved by Division.

10. If 9 lbs. of butter cost \$1.62, what will 3 lbs. cost?

11. If 32 cwt. cost £72, what cost 4 cwt.?

12. If 56 sheep cost £79 4s. what will 7 cost?

13. If the school tax on \$1673.12 is \$9, what will it be on \$418.28?

14. How long will 36 labourers take to dig a trench which 12 men can dig in 27 days?

15. A firm expended £190 17s. 6d. in 75 days, what will be the expenses for 25 days?

16. If 8 yards cost £4 12s., what will 13 yards cost?

ANALYSIS.—Here the quantity whose price is sought neither contains, nor is contained in, the quantity whose price is given, an even number of times. We therefore find the price of 1 yard as an intermediate step, the number 1 being in both quantities.

Thus, since 8 yds. cost £4 12s., 1 yd. cost $\frac{1}{8}$ of £4 12s.; and since 1 yd. cost $\frac{1}{8}$ of £4 12s., 13 yds. cost $\frac{13}{8}$ of £4 12s.; that is, £7 9s. 6d.

Such exercises are solved by Division and Multiplication combined.

17. If 7 articles cost 15s. 9d., what is the cost of 4?

18. If 11 tons of hay cost £37 9s. 10d., what is the cost of 8 tons?

19. If a man walk 21 miles in 7 hours, how far wil he walk in 9 hours?

20. A boy earns 5s 6d. in 3 days, in what time will he earn £9 18s.?

21. If 18 sheep are worth 3 cows, how many sheep are worth 21 cows?

22. What will 34 sheep cost at the rate of £368 2s. 9d. for 153 sheep?

23. If 18 lbs. of rice cost $67\frac{1}{2}$ cents, how many pounds can be purchased for 13s. 6d.?

24. How many yards of cloth may be had for $64.80, when 12 yards cost £3 12s.?

166. In the preceding exercises we found what multiple or part the quantity whose price was given, or the price whose quantity was given, was of that required,—and multiplied the remaining term by the result.

Thus, in the first exercise, dividing 20 by 5, we obtain 4 as quotient, then multiplying £9 by 4, we have £36 for the answer.

The question might have been asked thus:—

What sum of money will contain £9 as often as 20 yards contains 5 yards? Ans. £36

The number of times that one number is contained in another is called the *ratio* of the two numbers; thus the ratio of 5 to 20 is 4, and of 9 to 36 is 4.

167. Ratio is the comparison with each other of two numbers of the same kind.

168. The **Terms** are the two numbers compared.

169. The **Antecedent** is the *first* term.

170. The **Consequent** is the *second* term.

171. Ratio is expressed in two ways—

1st.—By placing two points, or a colon (:) between the numbers compared, writing the divisor before the points, and the dividend after the points. Thus, the ratio 5 to 7 is written 5 : 7; the ratio of 6 to 12 is written 6 : 12.

2nd.—In the form of a fraction. Thus, the ratio of 8 to 7 is $\frac{7}{8}$; the ratio of 5 to 9 is $\frac{9}{5}$.

NOTE.—In British publications the antecedent is put for the numerator and the consequent for the denominator; but the above form, which is that used in France, and in many parts of the United States, is more readily understood by beginners, because the *first term* of a proportion is always used as a divisor. It also renders the inversion of the fraction unnecessary when that form of ratio is used.

172. A **Simple Ratio** consists of a single couplet as 4 : 12.

173. A **Compound Ratio** is the product of two or more simple ratios. Thus,

The simple ratio of 4 to 8 is 2
The simple ratio of 3 to 9 is 3

The compound ratio of these is 12 to 72 6

174. In comparing numbers with each other, they must be of the same kind, and of the same denomination. Thus, shillings have a ratio to shillings. A foot has a ratio to a yard; for one is *three times* as long as the other; but a foot has not properly a ratio to an hour, for one cannot be said to be *longer* or *shorter* than the other.

NOTE.—When questions are solved by a direct application of the elementary rules, they are said to be worked by analysis. In the case of the previous exercises, it is merely finding the ratio of the two given terms of the same name, and multiplying the third by the result.

Exercises for the Slate.

1. What is the ratio of 3 to 27? Ans. 9
2. What is the ratio of 32 to 8? $\frac{1}{4}$
3. What is the ratio of 4 oz. to 3 lbs.?

Ans. 4 oz. : 3 lbs. = 4 oz. : 48 oz. = $\frac{1}{12}$

Required the ratios of the following numbers—

1.	7 to 14	5.	6 lbs. to 18 lbs.	9.	20 ft. to 40 yds.
2.	9 to 36	6.	28 lbs. to 4 lbs.	10.	60 m. to 4 fur.
3.	108 to 18	7.	9 oz. to 63 lbs.	11.	45 bus. to 3 qts.
4.	136 to 17	8.	17 yds. to 68 yds.	12.	3s. to 15 shillings

13. Which is the greater, the ratio of 9 to 63, or that of 8 to 72?

14. Which is the greater, the ratio of 120 to 85, or that of 240 to 170?

15. What is the ratio compounded of 8 : 10 and 20 : 16? Ans. 1

16. What is the ratio compounded of 35 : 40, and 60 : 75 and 21 : 19? Ans. $\frac{190}{147}$

17. What is the ratio of 19 lbs. 5 oz. 8 dwts. to 58 lbs. 4 oz. 4 dwts. Ans. $\frac{1}{3}$

18. If the antecedent be $\frac{3}{5}$ and the ratio $\frac{1}{6}$, what is the consequent? Ans. $\frac{1}{10}$

19. If the antecedent be 14.5 and the ratio 3, what is the consequent? Ans. 43.5

20. What sum of money will contain £6 10s. as often as 32 yards contain 8 yards? Ans. £26

21. How many acres of land will have the same ratio to 7 ac., that £16 has to £112? Ans. 49 ac.

22. How many yards of cloth will have the same ratio to 3 yds. 2 qrs. 2 nls., that £2 16s. 3d. has to £9 16s. $10\frac{1}{2}$d.?

Ans. 12 yds. 2 qrs. 3 nls.

23. What number compared with 8 will form a ratio equal to that of 4 to 6? Ans. 12

175. When the ratio of two numbers is *equal* to that of two other numbers, they are said to be proportional. Thus, the ratio of 4 to 6 is equal to the ratio of 8 to 12; and the four numbers are on that account said to be *proportional*, or to form a *simple proportion.*

176. Proportion is usually indicated by placing a double colon (: :) between the two ratios. Thus, 4 : 6 : : 8 : 12, and are read, As 4 is to 6 so is 8 to 12.

177. Since each ratio consists of two terms, every proportion must consist of at least *four terms.*

178. The **Extremes** are the first and fourth terms. The **Means** are the second and third terms.

179. In every proportion the product of the extremes is equal to the product of the means. Thus, in the proportion 4 : 8 : : 5 : 10 we have $4 \times 10 = 5 \times 8$.

180. From the preceding principles and illustrations, it follows that, any three terms of a proportion being given, the fourth may readily be found by the following

RULE. I. Divide the product of the extremes by one of the means, and the quotient will be the other mean. Or,

II. Divide the product of the means by one of the extremes, and the quotient will be the other extreme.

NOTE.—When the first and second terms are not both of the same name they must be reduced. The fourth term is always the same as the third term.

Exercises for the Slate.

Find the term not given in each of the following proportions:

1. 48 : 20 : : ( ) : 50. Ans. 120
2. 42 : 70 : : 3 : ( ). 5
3. 16 : 129 : : 112 : ( ). 903
4. 48 yd. : ( ) : : \$67.25 : \$201.75. 144 yd.
5. 17 yd. : 221 yd. : : ( ) : £1 1s. $11\frac{1}{4}$d. 1s. $8\frac{1}{4}$d.
6. ( ) : 160 yd. : : 8s. $5\frac{1}{4}$d. : 13s. 6d. 100 yd.
7. 3s. $4\frac{1}{2}$d. : ( ) : : 17 yd. : : 187 yd. £1 17s. $1\frac{1}{2}$d.
8. $\frac{5}{16}$: ( ) : : $\frac{1}{3}$: $\frac{2}{5}$. $\frac{3}{8}$

SIMPLE PROPORTION.

181. Simple Proportion is an equality of two simple ratios, and consists of four terms, any three of which being given, the fourth may readily be found.

EXAMPLE 1.—If 8 yds. of cloth cost $96, how much will 20 yds. cost at the same rate?

OPERATION.

```
      yd. yd.
As,   8 : 20 : : $96
                  20
                ————
               8)1920
                ————
                $240 Ans.
```

ANALYSIS.— Since 8 yards have the same ratio to 20 yds. as $96, the cost of the former has to the cost of the latter, we have the first three terms of a proportion given, namely one of the *extremes* and the *two means.* Now to arrange the given numbers in the order of a proportion, or *state the question*, we make $96 the *third* term, because it is of the same kind, and has the same ratio to the required answer, or fourth term, as the first has to the second. From the nature of the question, since the answer will be more than $96, or the third term, the *second* term must be larger than the *first*; we therefore put 20, the larger number, for the *second* term, and 8, the smaller, for the *first* term, and then the product of the means divided by the given extreme, gives the required extreme. **(180.)**

EXAMPLE 2.—If 50 men consume a certain quantity of flour in 20 days, how long would it take 35 men to consume a like quantity?

OPERATION.

```
    men   men    days
As  50  :  35  : :  20
           20
          ———
       50)700
          ———
           14  Ans.
```

OR,

$$\text{As } \cancel{50} : \cancel{35} :: \cancel{20}$$
$$\cancel{10} \quad 7 \quad 2$$

14 as before.

ANALYSIS.—Having stated the question as in the last example, we perceive that the first and second terms have a common factor, 5, we therefore cancel it, which leaves 10 and 7 as the new ratio. Again the factor 10 is common to the first and last terms, and we cancel it also, then multiplying 7 by 2 we have the answer as before.

Exercises for the Slate.

NOTE.—The pupil should write out each of the following exercises in words which will embrace the given *terms*. This will greatly facilitate his progress, and render him familiar with one of the most important agents of the science of calculation.

1. 13 yds. : 143 yds. : : 3s. $4\frac{1}{2}$d. : Ans. £1 17s. $1\frac{1}{2}$d.
2. 39 yds. : 432 yds. : : £1 1s. $11\frac{1}{4}$d. : £12 3s. 0d.
3. 8s. $5\frac{1}{4}$d. : 13s. 6d. : : 50 yds. : 80 yds.
4. 13s. 6d. : £2 17s. $4\frac{1}{2}$d. : : 68 yds. : 289 yds.
5. 48 men : 12 men : : 20 days : 5 days.
6. 5 bu. : 470 bu. : : £3 3s. : £296 2s. 0d.
7. 136 cwt. : 51 cwt. : : $3.80. 15s. $2\frac{1}{4}$d.
8. £13 18. $5\frac{1}{4}$d. : £95 8s. $6\frac{3}{4}$d. : : 165 tons : 1131 tons.
9. 144 days : 89 days : : £60 15s. : £37 10s. $11\frac{1}{4}$d.
10. $41.87 : £58 19s. $6\frac{3}{4}$d. : : 34 years. : 233 years.
11. 9 ac. 2 ro. 38 po. : 14 ac. 2 ro. 17 po. : : $8.45. Ans. $$12.67\frac{1}{2}$
12. 27 ac. 1 ro. 8 po. : 16 ac. 3 ro. 24 po. : : £22 3s. $7\frac{1}{2}$d. : Ans. $66.83
13. £14 6s. $11\frac{3}{4}$: $$27.92\frac{1}{2}$: : 19 yds. 2 qrs. 3 nls. Ans. 7 yds. 3 qrs. 2 nls.
14. 2 days : 3 years : : $1.10 : Ans. £124 6s. $6\frac{3}{4}$d.
15. 6 weeks : 68 years : : £4 15s. $4\frac{1}{2}$: Ans. £2810 7s. 8d.
16. 2 oz. 3 dwt. 21 grs : 4 oz. 17 dwt. 18 grs, : : £1 2s. $9\frac{1}{2}$d. Ans. $11.09

182. From the preceding illustrations and principles, we deduce the following general

RULE. I. Write for the third term that number which is of the same name as the required fourth term.

II. Of the other two numbers, write the larger for the second term, and the smaller for the first, when the answer should exceed the third term; but write the less for the second term, and the greater for the first, when the answer should be less than the third term.

III. Multiply the second and third terms together, and divide their product by the first.

NOTE.—*To shorten the work factors common to the first and second terms, or to the first and third terms, may be cancelled.*

Exercises for the Slate.

1. If I get 60 yards of cloth for $$486.66\frac{2}{3}$, how many yards will I get for £40? Ans. 24 yards.

2. If 36 men earn $192 in a week, what will 72 men earn in the same time? Ans. $384

3. If a railway train can run 525 miles in 15 hours, how far would it run in 7 hours? Ans. 245 miles.

4. If a grass field maintain 34 cows for 6 months, how long will it maintain 51 cows? Ans. 4 months.

5. If 17 cwt. be bought for £14, how many may be bought for \$116.80? Ans. 29 cwt. 16 lbs.

6. If 59 cwts. cost \$196, how many cwt. may be bought for \$140? Ans. 42 cwt. 16 lbs.

7. A silversmith pays £144 for 19 lbs. of silver, how much ought he to get for £234? Ans. 30 lbs. 10 oz. 10 dwt.

8. A lump of gold weighing 154 oz. costs \$2258.14, what will be the weight of a nugget which costs £290?

Ans. 96 oz. 5 dwt.

9. I bought 24 cwt. of sugar at £52 16s., required the price of 16 cwt.? Ans. £35 4s.

10. The wages of 6 men amount to \$18, required the wages of 9 men? Ans. \$27

11. Three score of sheep cost £66 16s. 8d., what will 36 sheep cost? Ans. \$195.16

12. A truckman charges \$15.47½ for 84 miles, how much is that for 56 miles? Ans. £2 11s. 7d.

13. If 4½ yds. cost £2 16s. 3d., what will 9 yds. cost at the same rate? Ans. \$27.38

14. A snail travels at the rate of 16 po. 2 yds. 2 ft. 9 in. in 3 hours, how far will he have gone in 2 days, travelling night and day? Ans. 6 fur. 24 po. 2 yds. 2 ft.

15. A school-room containing 120 pupils is 92 yds. 2 ft. in area, how much is that for each pupil? Ans. 6 ft. 132 in.

16. If 24¾ barrels of fish cost 39.27½, what will 8¼ barrels cost? Ans. \$13.09⅙

17. If 2¾ tons of coal cost \$13.33, required the price of 19¼ tons? Ans. £19 1s. 6d.

18. A person saves each week as much money as buys a square pole of ground, in what time will he be able to purchase a farm containing 21 ac. 7 po.? Ans. 64 yrs. 39 wks.

19. If 2 yds. 2 qrs. cost 16s. 7½d., what will 12 yds. 2 qrs. cost? Ans. 20.23

20. A boy who lives 455 yds. from the school goes to it in 6 min. 30 sec., how long would he take to go, if he were 2 miles 6 fur. 26 po. 1 yd. from it? Ans. 1 h. 11 min. 12 sec.

21. A chest of tea weighing 3 qrs. 22 lb. 15 oz., long wt., cost \$121.43, what will 5 chests, each weighing 1 qr. 27 lbs. 13 oz. cost? Ans. £65 2s. 3½d.

22. If a man mow 6 ac. 2 ro. 36 po. of barley in 5 days 8 hours, working 10 hours a day, in what time would he mow 16 ac. 8 ro. 10 po.? Ans. 14 da. 5 ho.

23. If 13 cwt. 0 qr. 9 lbs., long weight, cost £22 14s. $5\frac{3}{4}$d., what will 20 cwt. 3 qrs. 20 lb. cost? Ans. £36 7s. 2d.

24. A farmer draws a net profit of £23 17s. $2\frac{1}{4}$d. from 2 ac. 17 po.; how much should he receive at the same rate from 38 acres 3 ro. 32 po.? Ans. $2147.28

25. If $8\frac{3}{4}$ bushels of corn cost $4.20, what will be the cost of $13\frac{1}{2}$ bushels at the same rate? Ans. $6.48

26. If $1\frac{3}{4}$ yds. of cotton cloth cost $\$0.10\frac{6}{12}$, how many yds. can be bought for $100? Ans. $16\frac{2}{3}$ yds.

27. If $15\frac{5}{8}$ bu. of clover seed cost $$156\frac{1}{4}$, what will 9 bu. 2 pk. $2\frac{2}{5}$ qt. cost? Ans. $95.75

28. If $\frac{7}{8}$ of a barrel of apples cost $\$\frac{9}{11}$, how many can be bought for $\$\frac{60}{77}$? Ans. $\frac{5}{6}$ of a barrel.

29. A butcher selling meat sells $14\frac{1}{16}$ oz. for a pound; how much does he cheat a customer who buys of him to the amount of $30? Ans. $\$2.46\frac{3}{12}$

30. If I pay $6 for the loan of $100 for 1 year, what should I pay for $493? Ans. $29.58

31. If I borrow $2000, and keep it 1 year 4 mo., how long should I lend $240 as an eqivalent for the favour?
Ans. 2 yr. $9\frac{1}{3}$ mo.

32. If $\frac{3}{4}$ of $\frac{5}{6}$ of 4 ac. cost $\frac{1}{7}$ of $\frac{5}{12}$ of $140, what is the cost of 11 acres? Ans. $\$36\frac{2}{3}$

33. If I pay $$4\frac{1}{8}$ to a person for buying $100 worth of goods for me, what should I pay for buying $189.75 worth?
Ans. $\$7.82\frac{3}{4}$ nearly.

34. If a merchant makes a reduction of 1 penny in each shillings' worth of goods sold, how much is that in £100?
Ans. £8 6s. 8d.

35. An insolvent debtor fails for $2000, of which he is able to pay only $860, how much is that in each dollar, and how much will a person receive whose claim is $900?
Ans. $0.43 and $387

36. If £100 gain £3 in one year, what will £256 10s. 6d. gain in the same time? Ans. £7 13s. 11d. nearly.

37. Find the interest of £126 for one year at £5 per cent.
Ans. £6 6s.

NOTE.—In this exercise there are apparently only two terms. £5 per cent, however, just means £5 for £100. The above may therefore be written thus:—

If £100 gain £5 in one year, how much will £126 gain in the same time?

38. Find the interest of £126 14s. 6d. for 1 year at $8\frac{1}{3}$ per cent.

OPERATION.

£126	14	6 at $8\frac{1}{3}$	
		$8\frac{1}{3}$	
42	4	10	
1013	16	0	
£ 10,56	0	10	
20			
11,20			
12			
2,50			
2			
1,00			

£10 11s. $2\frac{1}{2}$d., Ans.

ANALYSIS.—Here, and in all similar cases, the first term being 100, we make no formal statement but merely multiply the second term by the third and divide by 100 as in **50.**

OR,

£	£	S.	D.	£
As $\cancel{100}$:	126	14	6 : :	$\cancel{8}\frac{1}{3}$
12				
£126	14	6 $\div$ 12 =		
£10	11	$2\frac{1}{2}$ as before.		

Here the third term is contained exactly 12 times in 100, we therefore cancel it. Dividing the second term by 12 we obtain the answer.

39. Find the interest of $186 for 1 year at 8 per cent.

OPERATION.

$	$	$
As $\cancel{100}$:	186 : :	$\cancel{8}$
1	.08	.08

$14.88 Ans.

ANALYSIS. — Here, dividing the first and third terms by 100 we have the quotients 1 and .08. We therefore multiply the second term by .08, and obtain the required interest. In a similar manner we may find the interest for one year at any given per cent.

Write out and solve the following exercises—

40. Find the interest of £186 10s. for 1 year at $6\frac{1}{4}$ per cent. Ans. £11 13s. $1\frac{1}{2}$d.

41. At $5\frac{1}{8}$ per cent., what is the interest of £196 16s. 8d. for 1 year? Ans. £10 1s. $9\frac{1}{2}$d.

42. Find the interest of \$196.78 for $8\frac{1}{2}$ per cent. for 1 year. Ans. \$16.72$\frac{1}{2}$ nearly.

43. What is the interest for 12 months of \$1836 at 6 per cent? Ans. \$110.16

44. What is the interest of \$1234.87$\frac{1}{2}$ for 1 year at $7\frac{1}{8}$ per cent? Ans. \$87.98$\frac{1}{2}$

45. Borrowed \$500.10 for 3 months, at 7 per cent; what will be the interest? Ans. \$8.75$\frac{1}{6}$

46. Gave a note for \$88.96 due in $2\frac{1}{2}$ years, at $6\frac{1}{4}$ per cent; what will be the interest? Ans. \$13.90

47. Borrowed \$988.65 for 2 years and 9 months, at 6 per cent; what will be the interest? Ans. \$163.12725

COMPOUND PROPORTION.

183. Compound Proportion is an equality between a *compound* ratio and a *simple* one.

$$\left.\begin{matrix}\text{Thus } 6 : 3 \\ \text{Into } 4 : 2\end{matrix}\right\} :: 12 : 3$$

$$\text{That is } 6 \times 4 : 3 \times 2 :: 12 \times 3 \text{; for } \overset{\text{extremes.}}{6 \times 4 \times 3} = \overset{\text{means.}}{12 \times 3 \times 2}$$

NOTE.—Compound proportion is chiefly applied to the solution of questions which would require *two or more statements* in simple proportion.

EXAMPLE 1.—If 8 men can reap 32 acres in 6 days, how many acres can 12 men reap in 24 days?

STATEMENT.

$$\left.\begin{matrix}\text{As } 8 \text{ men} : 12 \text{ men} \\ 6 \text{ days} : 15 \text{ days}\end{matrix}\right\} :: 32 \text{ ac.}$$

ANALYSIS.—In this example it is supposed that 8 men can reap 32 acres in 6 days; this being the case, it is asked or demanded how many acres 12 men can reap in 15 days. The question may therefore be divided into two parts, *supposition* and *demand.*

In order to state the question in the form of a proportion, we take from the supposition that quantity, 32 acres, which is of the same kind as the answer required, and place it for the third term. Then, taking the next number, 8 men, in the supposition, and 12 men, the corresponding number in the demand, and considering these with reference to the third term *only*, as in simple proportion, we find the answer is to exceed

the third term, and therefore place 12 men for the second term and 8 for the first. Again, comparing the remaining quantity, 6 days, in the supposition with the corresponding quantity, 15 days, in the demand with reference to the third term, 32 acres, we observe that if the time be increased the number of acres will also be increased; we therefore place 15 days in the second term and the 6 days in the first, and the question is stated.

OPERATION.

$$\text{As } \left.\begin{array}{r} 8 : 12 \\ 6 : 15 \end{array}\right\} :: 32$$

$$\begin{array}{r} 48 : 180 \\ 32 \\ \hline 360 \\ 540 \\ \hline \end{array} \text{ acres.}$$

$$\begin{array}{l} 48)5760(120 \text{ Ans.} \\ \quad 48 \\ \hline \quad\;\; 96 \\ \quad\;\; 96 \\ \hline \quad\;\;\; 0 \end{array}$$

ANALYSIS.—Since the product of the antecedents has the same ratio to the product of the consequents, as 32 has to the answer, (Art. **173**), we multiply 8 by 6 and 12 by 15 to form a simple ratio. The remainder of the work is the same as simple proportion.

EXAMPLE 2.—If 12 horses can plough 11 acres in 5 days, how many horses can plough 33 acres in 18 days?

Dividing the question into supposition and demand we have

$$\left.\begin{array}{l} 12 \text{ horses} \\ 11 \text{ acres} \\ 5 \text{ days} \end{array}\right\} \text{Supposition.} \qquad \left.\begin{array}{l} ? \\ 11 \text{ acres} \\ 18 \text{ days} \end{array}\right\} \text{Demand.}$$

$$\text{As } \left.\begin{array}{l} 11 \text{ acres} : 33 \text{ acres} \\ 18 \text{ days} : \;\; 5 \text{ days} \end{array}\right\} :: 12 \text{ horses.}$$

$$198 : 165$$

$$\frac{165 \times 12}{198} = 10 \text{ horses.}$$

Stating and working as in the former example we obtain 10 horses for the answer.

BY CANCELLATION.

$$\text{As } \left.\begin{array}{r} \cancel{11} : \overset{3\;\;1}{\cancel{33}} \\ \cancel{6}\;\cancel{18} : 5 \end{array}\right\} :: \underset{2}{\cancel{12}}$$

$$5 \times 2 = 10 \text{ as before.}$$

Here 11 is a common factor of the first and second terms, we therefore cancel it. Again, 3 being a common factor of 3 and 18, we divide each (3 and 18) by it, and set down the

quotients 1 and 6. For similar reasons we omit 6 and write 2 instead of 12. We then multiply 5 and 2 together and find the answer as before.

From these examples and illustrations we have the following

RULE. I. Take from the supposition that number which is of the same kind as the answer required, and place it for the third term.

II. Take the remaining numbers in pairs, one from the supposition and a corresponding one from the demand, and arrange them as in Simple Proportion.

III. Finally, multiply together all the second and third terms, divide the result by the product of the first terms, and the quotient will be the fourth term or answer.

NOTE. –When the first term has factors which are common to the second or third terms, *cancel the factors which are common, then divide the product of those remaining in the second and third terms by the product of those remaining in the first, and the quotient will be the answer.*

Exercises for the Slate.

1. If 18 masons can build a wall 120 feet long in 3 days, in what time will 24 men build a wall 480 feet long? Ans. 9 days.

2. If the wages for 8 men for 12 days be \$64, what will be the wages of 10 men for 6 days? Ans: \$40

3. If \$100 gain \$4 of interest in 12 months, how much will \$60 gain in 15 months? Ans. \$3

4. If £100 gain £5 of interest in 10 months, how much would £250 gain in 8 months? Ans. £10

5. The wages of 8 men for 4 days are \$19.50, what will be the wages of 12 men for 2 days? Ans. \$14.62½

6. If 12 reapers cut 71 ac. 2 ro. 8 po. in 6 days, how many acres will 8 reapers cut in 10 days? Ans. 79 ac. 2 ro.

7. If 16 horses in 9 days plough 110 acres, how many acres will 27 horses plough in 6 days. Ans. 123 ac. 3 ro.

8. If 208 families consume 6 cwt. of tea in 42 weeks, how much will 63 families consume in a year. Ans. $2\frac{1}{4}$ cwt.

9. If 18 men plant 29 ac. 2 ro. $26\frac{2}{3}$ po. of potatoes with the spade in 15 days, how many men would plant 17 ac. 3 ro. 8 po. in 6 days. Ans. 27 men.

10. If 69 yards of cloth 3 qrs. wide, make 24 pairs of trousers, how many pairs will 301 yds. 3 qrs. 2 nls., which is 1 yard wide, make? Ans. 140 pairs.

11. If a man walk 170 miles in 6 days, walking 15 hours a day, how many miles will he walk in 5 days, walking 12 hours a-day? Ans. 113 miles 2 fur. 26 po. $3\frac{2}{3}$ yds.

12. If 18 reapers cut 30 acres of barley in 6 days, working 10 hours a-day, how many reapers will it take to cut 40 acres in 4 days, working 12 hours a-day? Ans. 30 reapers.

13. If 16 men earn \$62.40 in 18 days, how many men will it take to earn \$140.40 in 24 days? Ans. 27 men.

14. If a family of 8 persons spend \$200 in 9 months, how much will 18 persons spend in 12 months? Ans. \$600

15. If 15 men working 12 hours a-day, can hoe 60 acres in 20 days, how long will it take 30 boys working 10 hours a-day, to hoe 96 acres, 6 men being equal to 10 boys? Ans. 32 days.

16. If 125 men can make an embankment 100 yards long, 20 feet wide, and 4 feet high in 4 days, working 12 hours a-day, how many men must be employed to make an embankment 1000 yards long, 16 feet wide, and 6 feet high, in 3 days, working 10 hours a-day? Ans. 2400 men.

17. A log of wood 60 feet long, 4 broad, 2 thick, cost \$128, what would be the price of one 45 feet long, $3\frac{1}{2}$ broad, and $2\frac{3}{4}$ thick? Ans. \$115.50.

18. If $42\frac{1}{2}$ yards of cloth, which is 18 in. wide, cost \$238.83$\frac{1}{3}$, what will $118\frac{1}{4}$ yards of yard-wide cloth of the same quality cost? Ans. \$1329.04.

19. If 400 men can make a canal which is to be a mile long, 40 feet broad, and 12 feet deep, in 20 days, working 8 hours a day, what length of canal, 30 feet wide and 16 deep, could 300 men make in 45 days, working 10 hours a day?
Ans. 2 miles 35 po.

20. Forty men engaged to finish a road, which was to be a mile long, in 60 days, but after three-fourths of it was done they left off. How many men would it take to finish the remainder in 6 days? Ans. 100 men.

21. If 5 horses require as much oats as 8 ponies, and 120 bushels last 12 ponies for 64 days, how long may 25 horses be kept for \$165 when oats are selling at \$0.55 per bushel?
Ans. 48 days.

22. If \$250 gain \$30 in 2 years, what will be the interest of \$750 for 5 years? Ans. \$225

23. If \$100 gain \$5 in 1 year, what will be the interest of \$575 for $3\frac{1}{2}$ years? Ans. 100.62$\frac{1}{2}$

24. What will be the interest of £125 for 4 years, if £150 will gain £10 10s. in 1 year? Ans. £35

25. If £100 gain £3 10s. in 1 year, what will £375 gain in 3 years and 8 months? Ans. £48 2s. 6d.

26. If \$100 gain \$4.50 in 1 year, what \$426.66$\frac{2}{3}$ gain from June 15th, 1865, to Sept. 18th, 1865? Ans. \$4.99

27. If £100 gain £4 in 365 days, what will be the gain on £690 10s. 6d. for 85 days? Ans. £6 8s. 7$\frac{1}{2}$d.

28. Find the interest of \$2737.50 for 56 days at 3$\frac{1}{2}$ per cent. Ans. \$14.70

NOTE.—The pupil may suppose that the full number of terms are not given in this exercise: but it will be readily seen that 3$\frac{1}{2}$ per cent is in reality \$3$\frac{1}{2}$ for the loan or interest of \$100 for one year or 365 days. The above question may be written thus:—

If \$100 gain \$3$\frac{1}{2}$ in 365 days. how much will \$2737.50 gain in 56 days?

NOTE.—The terms *per cent*, *interest*, *&c.*, have not been explained in the preceding pages: but as the illustrations of per centage in general depend on proportion, the pupil should, at this stage, be made acquainted with the principles involved. This will enable him to solve almost every question relating to per centage without considering them under any special rule.

Write out and solve the following exercises—

29. Find the interest of £812 6s. 8d. for 7 years 3 months at 5 per cent. Ans. £294 9s. 5d.

30. Lent \$2400 for 4 months, and received \$24.60 for interest; what was the rate per cent? Ans. 3.07$\frac{1}{2}$

31. Find the interest of \$3311.50 for 292 days at 2$\frac{1}{2}$ per cent. Ans. \$66.23

32. What is the interest of £660 for 8 months at 4$\frac{1}{2}$ per cent? Ans. £19 16s.

33. The value of a share in a railway is \$300, and the half-yearly dividend is \$16.80; required the rate per cent?
Ans. 11$\frac{1}{5}$ p. c.

34. Bought \$6000 worth of goods, and at the end of 70 days sold them for \$6200, what was the gain per cent?
Ans. 17$\frac{8}{21}$ p. c.

35. A person having borrowed a certain sum of money at 5 per cent., at the end of 3 months paid \$15, the amount of interest then due; how much did he borrow? Ans. \$1200

36. A person having mortgaged his property, pays \$40 of interest every three months; for what amount was the mortgage drawn, interest being charged at 6 per cent?
Ans. \$2666.66$\frac{2}{3}$

37. Dec. 18th, 1865—I borrowed \$6866.46. with which I purchased flour at \$6.66 a barrel. March 17th, 1866—I sold the flour for \$7.37$\frac{1}{2}$ a barrel, cash. How much did I gain by the transaction, interest being reckoned at 6 per cent?
Ans. \$636.71$\frac{1}{3}$

PERCENTAGE.

184. Per Cent. is a term derived from the Latin words *per centum*, and signifies *by the hundred*, or *hundredths*, that is, a certain number of parts of each *one hundred* parts, of whatever denomination. Thus, by 4 per cent., is meant \$4 of every \$100, 4 bushels for every 100 bushels, &c. Therefore, 4 per cent equals 4 hundredths $= .04 = \frac{4}{100} = \frac{2}{50} = \frac{1}{25}$. 8 per cent equals $.08 = \frac{8}{100} = \frac{2}{25}$.

185. Percentage is such a part of a number as indicated by the per cent.

186. The **Base** of percentage is the number on which the percentage is computed.

187. Since per cent. is any number of hundredths, it is usually expressed in the form of a *decimal;* but it may be expressed either as a *decimal* or a *common fraction* as in the following table.

NOTE.—In business, per cent is usually indicated by the sign %.

TABLE.

	Decimals.		Common fraction.		Lowest terms.
1 per cent.	= .01	=	$\frac{1}{100}$	=	$\frac{1}{100}$
2 per cent.	= .02	=	$\frac{2}{100}$	=	$\frac{1}{50}$
4 per cent.	= .04	=	$\frac{4}{100}$	=	$\frac{1}{25}$
5 per cent.	= .05	=	$\frac{5}{100}$	=	$\frac{1}{20}$
6 per cent.	= .06	=	$\frac{6}{100}$	=	$\frac{3}{50}$
7 per cent.	= .07	=	$\frac{7}{100}$	=	$\frac{7}{100}$
10 per cent.	= .1	=	$\frac{10}{100}$	=	$\frac{1}{10}$
$12\frac{1}{2}$ per cent.	= .125	=	$\frac{125}{1000}$	=	$\frac{1}{8}$

Exercises for the Slate.

1. Express decimally 3 per cent.; 4 per cent.; 6 per cent.; 9 per cent.; 11 per cent.; 15 per cent.; 20 per cent.; 25 per cent.; 130 per cent.; 375 per cent.

2. Express decimally $5\frac{1}{2}$ per cent.; $6\frac{1}{4}$ per cent.; $7\frac{1}{8}$ per cent.; $9\frac{1}{2}$ per cent.; $13\frac{1}{2}$ per cent.; $16\frac{1}{8}$ per cent.; $11\frac{5}{8}$ per cent.; $33\frac{1}{3}$ per cent.; $62\frac{1}{2}$ per cent.

3. Express decimally and vulgar fractionally $1\frac{1}{4}$ per cent. $2\frac{1}{2}$ per cent.; $25\frac{1}{2}$ per cent.

4. Express decimally $\frac{1}{4}$ per cent.; $\frac{3}{4}$ per cent.; $\frac{5}{8}$ per cent.

5. Express in the form of common fractions, in their lowest terms, 6 per cent.; 5 per cent.; $33\frac{1}{3}$ per cent.; $31\frac{1}{4}$ per cent.; 113 per cent.; $18\frac{5}{8}$ per cent.

CASE I.

188. *To find the percentage of any number.*

EXAMPLE.—A man having 125 bushels of wheat, sold 25 per cent. of that quantity, how much did he sell?

OPERATION.

$$\begin{array}{r} 125 \\ .25 \\ \hline 625 \\ 250 \\ \hline 31.25 = 31\frac{1}{4} \end{array}$$

ANALYSIS.—Since 25 per cent. is $\frac{25}{100} = .25$, he sold $.25 \times 125$ bus., or 125 bush. $\times .25 = 13\frac{1}{4}$ bhshels. Or, 25 per cent. is $\frac{25}{100} = \frac{1}{4}$, and $\frac{1}{4}$ of $125 = 31\frac{1}{4}$. Hence the following—

RULE. Multiply the given number or quantity by the rate per cent., expressed decimally, and point off as in decimals. Or,

Take such a part of the given number as the number expressing the rate is part of 100.

Exercises.

1. What is 5 per cent. of \$18940? Ans. \$947
2. What is $8\frac{1}{2}$ per cent. of \$1248? \$106.08
3. What is $7\frac{1}{4}$ per cent. of \$56.75? $\$4.11\frac{7}{16}$
4. What is $6\frac{3}{4}$ per cent. of 1967 bus.? 132.7725 bus.
5. What is $9\frac{4}{5}$ per cent. of 275 miles? 26.95 miles.
6. What is 25 per cent. of $\frac{5}{8}$?
   25 per cent. $= \frac{25}{100} = \frac{1}{4}$, and $\frac{5}{8} \times \frac{1}{4} = \frac{5}{16}$ Ans.
7. What is $\frac{1}{4}$ per cent. of \$2526.40? Ans. \$6.316.
8. What is $\frac{1}{3}$ per cent. of \$75,000? \$250.00
9. A farmer having 1500 sheep, sold 25 per cent. of them; how many did he sell? Ans. 375 sheep.
10. A merchant imported 1500 boxes of oranges, and $12\frac{1}{2}$ per cent. of them decayed; how many boxes did he lose, and how many had he left? Ans. 187.5 lost.
    1312.5 saved.

CASE II.

189. *To find what per cent. one number is of another.*

EXAMPLE.—A man having purchased a horse for \$170, sold him for \$17 less; what per cent. of his money did he lose?

OPERATION.

$17 \div 170 = .10 = 10$ per cent.

OR,

$\frac{17}{170} = \frac{1}{10} = .10 = 10$ per cent.

ANALYSIS.—We multiply the base by the rate per cent. to obtain the percentage (188); conversely, we divide the per centage by the base to obtain the rate. Or, since \$170 is 100 per cent. of his money, \$17 is $\frac{17}{170}$, equal to $\frac{1}{10}$ of 100 per cent., which is 10 per cent. Hence the following—

RULE. Divide the per centage by the base, and the quotient will be the rate per cent., expressed decimally. Or, **Take such a part of 100 as the per centage is part of the base.**

Exercises for the Slate.

1. What per cent. of \$9876 is \$2469? Ans. 25
2. What per cent. of \$7656 is \$957? Ans. $12\frac{1}{2}$
3. What per eent. of 4 tons 16 cwt. is 3 tons. 12 cwt? Ans. 75 per cent.
4. What per cent. of 6 bushels 1 peck is 4 bushels 2 pecks 6 quarts? Ans. 75 per cent.
5. A man having 900 acres of land, sold $\frac{1}{3}$ of it at one time, and $\frac{1}{2}$ of the remainder at another time; what per cent. remained unsold? Ans. $33\frac{1}{3}$ per cent.

CASE III.

190. *To find a number when a certain per cent. of it is given.*

EXAMPLE.—A man sold $31\frac{1}{4}$ bushels of wheat, being 25 per cent. of all he had; how much had he at first?

OPERATION.

31.25 bushels $\div .25 = 125$

OR,

$$\frac{31\frac{1}{4}}{25} \times 100 = \frac{125}{100} \times 100 = 125$$

ANALYSIS.—We are here required to find the base, of which $31\frac{1}{4}$ bushels is the percentage.—Now, percentage equals base multiplied by the rate per cent.; conversely, base equals percentage divided by the rate per cent. Or, $31\frac{1}{4}$ bushels is 25 per cent. of all he had; $\frac{1}{25}$ of $31\frac{1}{4}$ bushels, or $\frac{125}{100}$ equals 1 per cent. of all he had, and 100 times $\frac{125}{100}$ equals 100 per cent. of all he had. Hence the following—

RULE. Divide the percentage by the rate per cent., expressed decimally, and the quotient will be the base, or number required. Or,

Take as many times 100 as the percentage is times the rate per cent.

Exercises for the Slate.

24 is 8 per cent. of what number ? *Ans.* 300

2. 42 is 7 per cent. of what number ? 600

3. $39\frac{1}{2}$ is 5 per cent. of what number ? 790

4. A man, owning 30 per cent. of a shoe factory, sells $33\frac{1}{3}$ per cent. of his share for $1111.275, what is the value of the whole factory ? *Ans.* 11112.75

APPENDIX I.

KEY TO THE SELF-TESTING EXERCISES.

ADDITION.

All the exercises given in this Rule as self-testing are formed as shown in section 3.

To test the sum of any number of rows or lines we may use any of the three following methods.

1st. As the first line of each exercise is a multiple of 9, the sum of any number of lines must also be a multiple of 9; therefore casting the 9's out of the sum, if the work is correct, there will be no excess.

If there be an error in any of the lines it may also be detected by casting out the 9's in the same manner.

2nd. If the exercise is composed of 5 rows, the sum of all the rows will be 12 times the first line. If composed of 6 rows it will be 20 times the first line, and so on as may be seen in the following examples.

(1)

1467	First line	=1 times	
1467	Second "	=1 "	
2934	Third "	=2 "	1st line.
4401	Fourth "	=3 "	"
7335	Fifth "	=5 "	"
17604	Sum	=12 times 1st line.	

(2)

1467	First line	=1
1467	Second "	=1
2934	Third "	=2
4401	Fourth "	=3
7335	Fifth "	=5
11736	Sixth "	=8
29340	Sum	=20 times 1st line.

3rd. The sum of a required number of lines added to the first line will be equal to the line that is *two* more than the required number of lines. Thus let 6 be the required number of lines. The sum of six lines added to the first line will be equal to the eighth line. Let 11 be the required number of lines. The sum of eleven lines added to the first line will give the 13th line.

EXAMPLE.—Find the sum of 162 extended to 8 rows, and test the result by the tenth line.

OPERATION.

1st line	162	
2nd "	162	
3rd "	324	
4th "	486	
5th "	810	
6th "	1296	
7th "	2106	
8th "	3402	
		8748 = sum of eight lines.
9th "	5508	162 = first line.
Tenth line	8910	8910 = line that is two more than

the required number of lines, *i. e.*, (8 × 2) 10th line.

NOTE.—As soon as the pupil fully understands the principles of addition he should be required to test his work as above, and thus facilitate his progress.

SUBTRACTION.

The exercises under this rule are to be worked by the pupil as shown in the following example.

18717	minuend.
12478	subtrahend.
6239	difference.
6239	difference betwen 2d and 3d line.

ANALYSIS.—We first take the subtrahend from the minuend, then this difference from the subtrahend. If the two last lines are alike, the work is correct.

MULTIPLICATION.

SECTION 1.—The test of the exercises in this section may be seen from the construction of each.

SECTION 2.—In the exercises in this section the teacher will observe that every line in the working, and every product, is a multiple of nine, and by adding the digits in any line or product he can ascertain if it is correct.

SECTIONS 3, 4 and 5.—The manner of testing the exercises in these sections may be readily seen from their construction.

DIVISION.

SECTION 1.—Each dividend is a multiple of its divisor, consequently, if worked correctly there will be no remainders.

SECTIONS 4 and 6.—In the exercises under these sections each dividend is a multiple of nine, also each divisor, and the remainders, if any, are divisible by 9, and each dividend is divisible by all the divisors given with remainders as above. These sections, therefore, contain 841 exercises.

ADDITION OF DECIMALS.

Increase each figure of the second line by unity, and prefix the first figure of the exercise. The effect of 9 occurring in the second line should be particularly noted.

NOTE.—The second line may be varied at pleasure.

SUBTRACTION OF DECIMALS.

Same as Simple Subtraction.

MULTIPLICATION OF DECIMALS.

Same as Section 3 of Simple Multiplication.

DIVISION OF DECIMALS.

The quotients are without remainders, and each is a multiple of 9.

REDUCTION DESCENDING.

The answers to all the exercises given in Reduction descending are to be tested by the sum of the digits, which, if correct, will be found to contain some multiple of 9 without any excess.

REDUCTION ASCENDING.

(1)

Reduce 15270 pence to pounds.

12)15270

2,0)127,2s. 6d.

£63 12 6

(2)

Reduce 311267 far. to pounds.

4)311267

12)77816¾d.

2,0)648,4s. 8d.

£324 4 8¾

(3)

Reduce 28197 dwts. to lbs.

2,0)28197

12)1409 17

117 lbs. 5 oz. 17 dwt.

For exercises like examples (1) and (2) test the pounds by the sum of the digits, then double the two right hand figures, calling the units pence, and the other figures shillings. Thus £36 7s. 2d. Here the number of pounds = 36. Test the pounds by the sum of the digits. Then $36 \times 2 = 72$, take 2 for pence, and 7 for shillings.

If, as in example (2) the answer contains three figures, and the left hand figure under four, then for pounds, shillings, and pence, the same test as before, and for farthings the same number as the left hand figure. Thus, in the example, the number of pounds is 324, which being tested by the sum of the digits, ($3 + 2 + 4 = 9$, leaving no excess). Then, $24 \times 2 = 48$, take 8 for pence and 4 for shillings.— The left hand figure is 3—take 3 for farthings.

In exercises like (3) the number in the highest denomination to be tested in the same way, and the same number of the lowest denomination taken. Thus, in the example the number of the highest denomination is 117 (test by the sum of the digits). Then the same number of the lowest denomination 117 dwt.. *i. e.* 5 oz. 17 dwt.

COMPOUND ADDITION.

Test exactly the same as in addition of decimals, with the exception that unity must be added, not to each figure, but to each denomination excepting farthings.

COMPOUND SUBTRACTION.

SECTION 1.—Same test as Reduction ascending.
SECTION 2.—May be seen in example worked.
The exercises under Division, and Practice are sufficiently explicit.

PROPORTION.

The answer, when in Simple Numbers, to be tested by the sum of its digits; and when Compound, the same as Reduction Ascending.

APPENDIX II.

TABLE I.

EQUIVALENT OF CANADA CURRENCY IN PENCE STERLING.

Dollars or Cents		
	1	*d.*493150684
	2	.986301369
	3	1.47945205
	4	1.97260273
	5	2.46575342
	6	2.95890410
	7	3.45205479
	8	3.94520547
	9	4.43835616

NOTE—
For any number of CENTS from 1 to 9, point as in the Table.
" " " 10 to 90 move the point 1 place to the right.

For	DOLLARS	$1 to $9	move the point	2	places	to the right
"	"	$10 to $90	"	3	"	"
"	"	$100 to $900	"	4	"	"
"	"	$1000 to $9000	"	5	"	"
"	"	$10,000 to $90,000	"	6	"	"
"	"	$100,000 to $900,000	"	7	"	"
"	"	$1,000,000 to $9,000,000	"	8	"	"

☞ In working exercises, if the the figures to the right of the point range from—

.13 to .38	reckon them	$\frac{1}{4}$d
.39 to .63	"	$\frac{1}{2}$d
.64 to .88	"	$\frac{3}{4}$d
.89 to .99	"	1d

EXAMPLES.—Convert the following amounts, Canada currency, to pounds, shillings and pence, stg:—(1) \$0.08; (2) \$0.10; (3) \$10; (4) \$100; (5) \$1,000; (6) \$10,000; (7) \$1,000,000.10; (8)\$225.55

(1) 8 cts=4d (2) 10 cts=5d

(3) \$10=12)493.15

2,0)4,1. $1\frac{1}{4}$

£2. 1s $1\frac{1}{4}$d

(4) \$100=12)4931.50

2,0)41,0. $11\frac{1}{2}$

£20. 10s $11\frac{1}{2}$d

(5) \$1,000=12)49315.

2,0)410,9. 7

£205. 9s 7d

(6) \$10,000=12)493150.68

2,0)4109,5. $10\frac{3}{4}$

£2054 15s $10\frac{3}{4}$d

(7) \$1,000,000.00 = 49315068.44
.10 = 4.93

\$1,000,000.10 12)49315073.37

2,0)410958,9. $5\frac{1}{4}$

£205479. 9s $5\frac{1}{4}$d

8) \$200.	=	9863. 01
20.	=	986. 30
5.	=	246. 57
.50	=	24. 65
.05	=	2. 46
\$225.55		12)11123. 99

2,0)92,6. 11

£46. 6s 11d

TABLE II.

EQUIVALENT OF POUNDS, SHILLINGS & PENCE STG., IN CANADA CURRENCY.

£		s.		d.	
1	$4.86666666	1	$.243	1	$.02
2	9.73333333	2	.486	2	.04
3	14.59999999	3	.729	3	.06
4	19.46666666	4	.973	4	.08
5	24.33333333	5	1.216	5	.10
6	29.19999999	6	1.459	6	.12
7	34.06666666	7	1.703	7	.14
8	38.93333333	8	1.946	8	.16
9	43.79999999	9	2.189	9	.18
		10	2.433	10	.20
		11	2.676	11	.22
		12	2.919	—	—
		13	3.163	f.	
		14	3.406	1	.005
		15	3.649	2	.010
		16	3.893	3	.015
		17	4.136		
		18	4.379		
		19	4.623		

NOTE—For shillings, pence and farthings, point as in the table
" POUNDS from £1 to £9 " "
£10 to £90 move the point 1 place to right
£100 to £900 " 2 places "
£1000 to £9000 " 3 " "
£10,000 to £90,000 " 4 " "
£100,000 to £900,000 " 5 " "
£1,000,000 to £9,000,000 " 6 " "

☞ If the mills reach 6 or over reckon them as 1 cent.

EXAMPLES.—Convert the following amounts, sterling money, to Canadian currency :—

(1) £1=$4.87
(2) £100=$486.67
(3) £1000=$4866.67
(4) £10,000=$48666.67
(5) £100,000=$486666.67
(6) £1,000,000=$4,866,666.67

(7) £4 10s 9½d
£4 = $19.466
10s = 2.433
9d = .18
2f = .01
$22.09

TABLE III.

EQUIVALENT OF FORMER CURRENCY OF NOVA SCOTIA IN CANADA CURRENCY.

N.S. Dollars or Cents	
1	$.97333333
2	1.94666666
3	2.92000000
4	3.89333333
5	4.86666666
6	5.84000000
7	6.81333333
8	7.78666666
9	8.76000000

NOTE—Move the point as in TABLE I. If the mills reach 6 or over reckon them as 1 cent.

EXAMPLES—Convert (1) \$20, (2) \$75, (3) \$4,120.10 former currency of Nova Scotia to Canadian currency :—

(1) \$20 = \$19.47

(2)	\$70 =	\$68.13
	5 =	4.87
	\$75	\$73.00

(3)	\$4,000	=	\$3,893.33
	100	=	97.33
	20	=	19.47
	.10	=	.10
	\$4,120.10		\$4,010.23

OF

or over

,120.10
:—

CONTENTS.

SIMPLE NUMBERS.

DECIMALS.

COMPOUND NUMBERS.

VULGAR OR COMMON FRACTIONS.